El-Hawary, M.E. "Basics of Electric Energy System Theory"
Electrical Energy Systems.
Series Ed. Leo Grigsby
Boca Raton: CRC Press LLC, 2000

Chapter 2

BASICS OF ELECTRIC ENERGY SYSTEM THEORY

2.1 INTRODUCTION

This chapter lays the groundwork for the study of electric energy systems. We develop some basic tools involving fundamental concepts, definitions, and procedures. The chapter can be considered as simply a review of topics utilized throughout this work. We start by introducing the principal electrical quantities.

2.2 CONCEPTS OF POWER IN ALTERNATING CURRENT SYSTEMS

The electric power systems specialist is in many instances more concerned with electric power in the circuit rather than the currents. As the power into an element is basically the product of voltage across and current through it, it seems reasonable to swap the current for power without losing any information in describing the phenomenon. In treating sinusoidal steady-state behavior of circuits, some further definitions are necessary. To illustrate the concepts, we will use a cosine representation of the waveforms.

Consider the impedance element $Z = Z\angle\phi$. For a sinusoidal voltage, υ(t) given by

$$\upsilon(t) = V_m \cos\omega t$$

The instantaneous current in the circuit is

$$i(t) = I_m \cos(\omega t - \phi)$$

where

$$I_m = V_m / |Z|$$

The instantaneous power is given by

$$p(t) = \upsilon(t)i(t) = V_m I_m[\cos(\omega t)\cos(\omega t - \phi)]$$

This reduces to

$$p(t) = \frac{V_m I_m}{2}[\cos\phi + \cos(2\omega t - \phi)]$$

Since the average of $\cos(2\omega t - \phi)$ is zero, through 1 cycle, this term therefore contributes nothing to the average of p, and the average power p_{av} is given by

$$p_{\text{av}} = \frac{V_m I_m}{2} \cos\phi \qquad \textbf{(2.1)}$$

Using the effective (rms) values of voltage and current and substituting $V_m = \sqrt{2}(V_{\text{rms}})$, and $I_m = \sqrt{2}(I_{\text{rms}})$, we get

$$p_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi \qquad \textbf{(2.2)}$$

The power entering any network is the product of the effective values of terminal voltage and current and the cosine of the phase angle ϕ, which is, called the *power factor* (PF). This applies to sinusoidal voltages and currents only. When reactance and resistance are present, a component of the current in the circuit is engaged in conveying the energy that is periodically stored in and discharged from the reactance. This stored energy, being shuttled to and from the magnetic field of an inductance or the electric field of a capacitance, adds to the current in the circuit but does not add to the average power.

The average power in a circuit is called *active power*, and the power that supplies the stored energy in reactive elements is call *reactive power*. Active power is P, and the reactive power, designated Q, are thus*

$$P = VI \cos\phi \qquad \textbf{(2.3)}$$

$$Q = VI \sin\phi \qquad \textbf{(2.4)}$$

In both equations, V and I are rms values of terminal voltage and current, and ϕ is the phase angle by which the current lags the voltage.

To emphasize that the Q represents the nonactive power, it is measured in reactive voltampere units (var).

---

* If we write the instantaneous power as

$$p(t) = V_{\text{rms}} I_{\text{rms}} [\cos\phi(1 + \cos 2\omega t)] + V_{\text{rms}} I_{\text{rms}} \sin\phi \sin 2\omega t$$

then it is seen that

$$p(t) = P(1 + \cos 2\omega t) + Q \sin 2\omega t$$

Thus P and Q are the average power and the amplitude of the pulsating power, respectively.

Figure 2.1 shows the time variation of the various variables discussed in this treatment.

Assume that V, $V\cos\phi$, and $V\sin\phi$, all shown in Figure 2.2, are each multiplied by I, the rms values of current. When the components of voltage $V\cos\phi$ and $V\sin\phi$ are multiplied by current, they become P and Q, respectively. Similarly, if I, $I\cos\phi$, and $I\sin\phi$ are each multiplied by V, they become VI, P, and Q, respectively. This defines a power triangle.

We define a quantity called the *complex* or *apparent power*, designated S, of which P and Q are components. By definition,

$$\begin{aligned} S &= P + jQ \\ &= VI(\cos\phi + j\sin\phi) \end{aligned}$$

Using Euler's identity, we thus have

$$S = VIe^{j\phi}$$

or

$$S = VI\angle\phi$$

It is clear that an equivalent definition of complex or apparent power is

$$S = VI^* \qquad \textbf{(2.5)}$$

We can write the complex power in two alternative forms by using the relationships

$$V = ZI \qquad \text{and} \qquad I = YV$$

This leads to

$$S = ZII^* = Z|I|^2 \qquad \textbf{(2.6)}$$

or

$$S = VY^*V^* = Y^*|V|^2 \qquad \textbf{(2.7)}$$

Consider the series circuit shown in Figure 2.3. Here the applied voltage is equal to the sum of the voltage drops:

$$V = I(Z_1 + Z_2 + \ldots + Z_n)$$

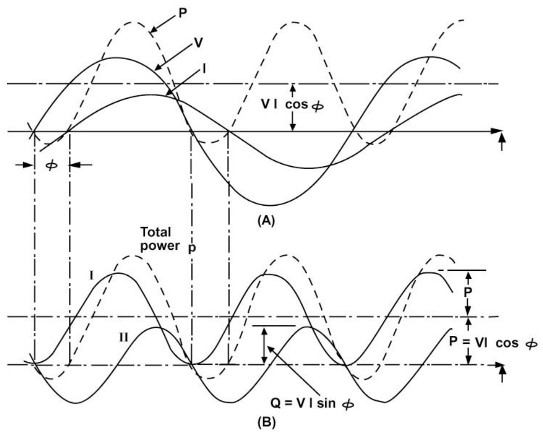


Figure 2.1 Voltage, Current, and Power in a Single-Phase Circuit.

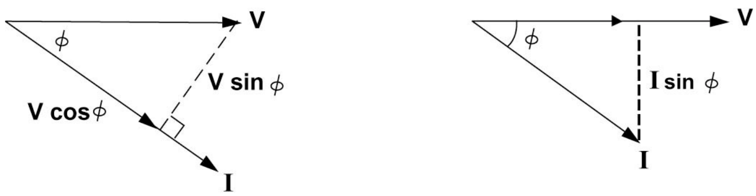


Figure 2.2 Phasor Diagrams Leading to Power Triangles.

Multiplying both sides of this relation by I^* results in

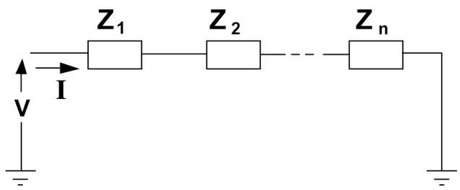
$$S = \sum_{i=1}^{n} S_i \tag{2.8}$$

with the individual element's complex power.

$$S_i = |I|^2 Z_i \tag{2.9}$$

Equation (2.8) is known as the summation rule for complex powers. The rule also applies to parallel circuits.

The phasor diagram shown in Figure 2.2 can be converted into complex power diagrams by simply following the definitions relating complex power to voltage and current. Consider an inductive circuit in which the current lags the voltage by the angle ϕ. The conjugate of the current will be in the first quadrant in the complex plane as shown in Figure 2.4(a). Multiplying the phasors by **V**, we obtain the complex power diagram shown in Figure 2.4(b). Inspection of the diagram as well as the previous development leads to a relation for the power factor of the circuit:



$$\cos\phi = \frac{P}{|S|}$$

Figure 2.3 Series Circuit.

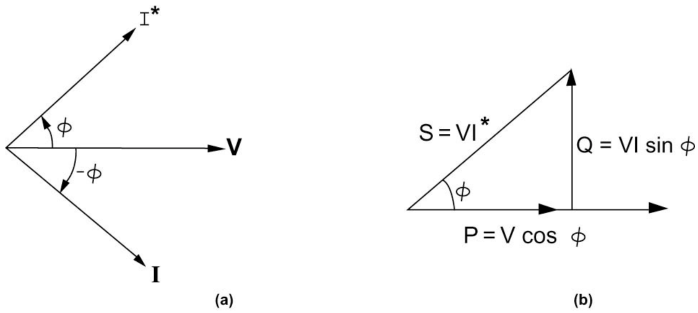


Figure 2.4 Complex Power Diagram

Example 2.1

Consider the circuit composed of a series *R-L* branch in parallel with capacitance with the following parameters:

$$R = 0.5 \text{ ohms}$$
$$X_L = 0.8 \text{ ohms}$$
$$B_c = 0.6 \text{ siemens}$$

Assume

$$V = 200\angle 0 \text{ V}$$

Calculate the input current and the active, reactive, and apparent power into the circuit.

Solution

The current into the *R-L* branch is given by

$$I_Z = \frac{200}{0.5 + j0.8} = 212\angle -57.99^\circ \text{ A}$$

The power factor (PF) of the *R-L* branch is

$$\begin{aligned} \mathrm{PF}_Z &= \cos\phi_Z = \cos 57.99^\circ \\ &= 0.53 \end{aligned}$$

The current into the capacitance is

$$I_c = j(0.6)(200) = 120\angle 90^\circ \text{ A}$$

The input current I_t is

$$\begin{aligned} I_t &= I_c + I_Z \\ &= 212\angle -57.99^\circ + 120\angle 90^\circ \\ &= 127.28\angle -28.01^\circ \end{aligned}$$

The power factor (PF) of the coverall circuit is

$$\mathrm{PF}_t = \cos\phi_t = \cos 28.01^\circ = 0.88$$

Note that the magnitude of I_t is less than that of I_Z, and that $\cos\phi$ is higher than $\cos\phi_Z$. This is the effect of the capacitor, and its action is called *power factor correction* in power system terminology.

The apparent power into the circuit is

$$\begin{aligned} S_t &= VI_t^* \\ &= (200\angle 0)(127.28)\angle 28.01^\circ \\ &= 25{,}456.00\angle 28.01^\circ \text{ VA} \end{aligned}$$

In rectangular coordinates we get

$$S_t = 22{,}471.92 + j11{,}955.04$$

Thus, the active and reactive powers are

$$\begin{aligned} P_t &= 22{,}471.92 \text{ W} \\ Q_t &= 11{,}955.04 \text{ var} \end{aligned}$$

2.3 THREE-PHASE SYSTEMS

The major portion of all electric power presently used in generation, transmission, and distribution uses balanced three-phase systems. Three-phase operation makes more efficient use of generator copper and iron. Power flow in

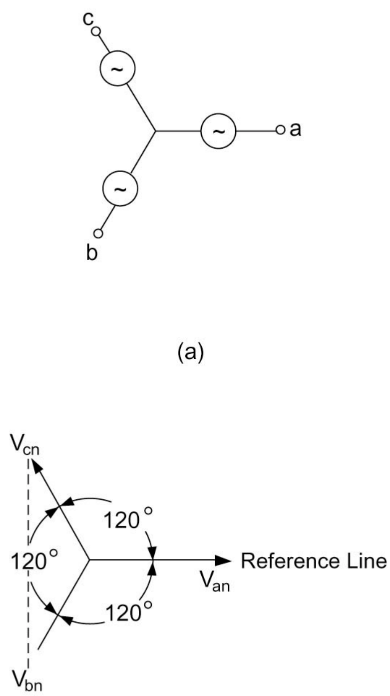


Figure 2.5 A Y-Connected Three-Phase System and the Corresponding Phasor Diagram.

single-phase circuits was shown in the previous section to be pulsating. This drawback is not present in a three-phase system. Also, three-phase motors start more conveniently and, having constant torque, run more satisfactorily than single-phase motors. However, the complications of additional phases are not compensated for by the slight increase of operating efficiency when polyphase systems other than three-phase are used.

A balanced three-phase voltage system is composed of three single-phase voltages having the same magnitude and frequency but time-displaced from one another by 120°. Figure 2.5(a) shows a schematic representation where the three single-phase voltage sources appear in a Y connection; a Δ configuration is also possible. A phasor diagram showing each of the phase voltages is also given in Figure 2.5(b).

Phase Sequence

As the phasors revolve at the angular frequency ω with respect to the reference line in the counterclockwise (positive) direction, the positive maximum value first occurs for phase *a* and then in succession for phases *b* and *c*. Stated in a different way, to an observer in the phasor space, the voltage of phase *a* arrives first followed by that of *b* and then that of *c*. The three-phase voltage of Figure 2.5 is then said to have the phase sequence *abc* (*order* or *phase*

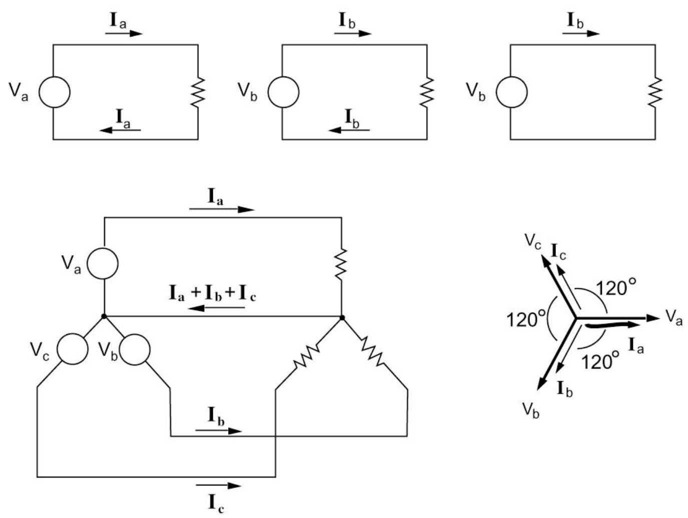


Figure 2.6 A Three-Phase System.

sequence or *rotation* are all synonymous terms). This is important for applications, such as three-phase induction motors, where the phase sequence determines whether the motor turns clockwise or counterclockwise.

With very few exceptions, synchronous generators (commonly referred to as *alternators*) are three-phase machines. For the production of a set of three voltages phase-displaced by 120 electrical degrees in time, it follows that a minimum of three coils phase-displaced 120 electrical degrees in space must be used.

It is convenient to consider representing each coil as a separate generator. An immediate extension of the single-phase circuits discussed above would be to carry the power from the three generators along six wires. However, instead of having a return wire from each load to each generator, a single wire is used for the return of all three. The current in the return wire will be $I_a + I_b + I_c$; and for a balanced load, these will cancel out. If the load is unbalanced, the return current will still be small compared to either I_a, I_b, or I_c. Thus the return wire could be made smaller than the other three. This connection is known as a four-wire three-phase system. It is desirable for safety and system protection to have a connection from the electrical system to ground. A logical point for grounding is the generator neutral point.

Current and Voltage Relations

Balanced three-phase systems can be studied using techniques developed for single-phase circuits. The arrangement of the three single-phase voltages into a *Y* or a Δ configuration requires some modification in dealing with the overall system.

Y Connection

With reference to Figure 2.7, the common terminal n is called the *neutral* or *star* (Y) *point*. The voltages appearing between any two of the line terminals a, b, and c have different relationships in magnitude and phase to the voltages appearing between any one line terminal and the neutral point n. The set of voltages V_{ab}, V_{bc}, and V_{ca} are called the *line voltages*, and the set of voltages V_{an}, V_{bn}, and V_{cn} are referred to as the *phase voltages*. Analysis of phasor diagrams provides the required relationships.

The effective values of the phase voltages are shown in Figure 2.7 as V_{an}, V_{bn}, and V_{cn}. Each has the same magnitude, and each is displaced 120° from the other two phasors.

Observe that the voltage existing from a to b is equal to the voltage from a to n (i.e., V_{an}) plus the voltage from n to b.

For a balanced system, each phase voltage has the same magnitude, and we define

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p \tag{2.10}$$

where V_p denotes the effective magnitude of the phase voltage.

We can show that

$$\begin{aligned} V_{ab} &= V_p(1 - 1\angle -120^\circ) \\ &= \sqrt{3}V_p\angle 30^\circ \end{aligned} \tag{2.11}$$

Similarly, we obtain

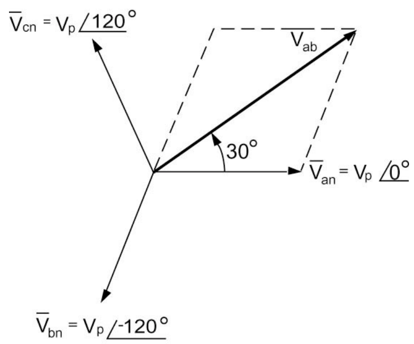


Figure 2.7 Illustrating the Phase and Magnitude Relations Between the Phase and Line Voltage of a Y Connection.

$$V_{bc} = \sqrt{3}V_p\angle -90^\circ \quad \textbf{(2.12)}$$

$$V_{ca} = \sqrt{3}V_p\angle 150^\circ \quad \textbf{(2.13)}$$

The line voltages constitute a balanced three-phase voltage system whose magnitudes are $\sqrt{3}$ times the phase voltages. Thus, we write

$$V_L = \sqrt{3}V_p \quad \textbf{(2.14)}$$

A current flowing out of a line terminal *a* (or *b* or *c*) is the same as that flowing through the phase source voltage appearing between terminals *n* and *a* (or *n* and *b*, or *n* and *c*). We can thus conclude that for a *Y*-connected three-phase source, the line current equals the phase current. Thus,

$$I_L = I_p \quad \textbf{(2.15)}$$

Here I_L denotes the effective value of the line current and I_p denotes the effective value for the phase current.

Δ Connection

Consider the case when the three single-phase sources are rearranged to form a three-phase Δ connection as shown in Figure 2.8. The line and phase voltages have the same magnitude:

$$|V_L| = |V_p| \quad \textbf{(2.16)}$$

The phase and line currents, however, are not identical, and the relationship

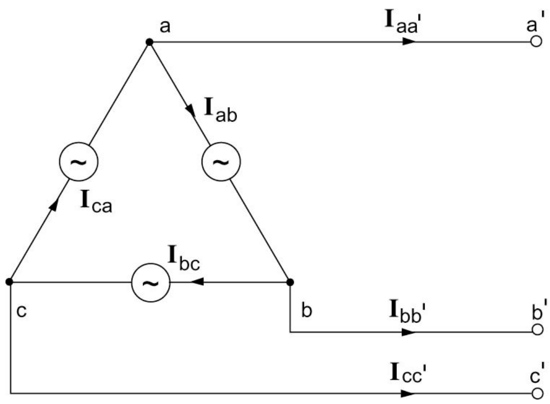


Figure 2.8 A Δ-Connected Three-Phase Source.

between them can be obtained using Kirchhoff's current law at one of the line terminals.

In a manner similar to that adopted for the *Y*-connected source, let us consider the phasor diagram shown in Figure 2.9. Assume the phase currents to be

$$I_{ab} = I_p \angle 0$$
$$I_{bc} = I_p \angle -120^\circ$$
$$I_{ca} = I_p \angle 120^\circ$$

The current that flows in the line joining a to a' is denoted $I_{aa'}$ and is given by

$$I_{aa'} = I_{ca} - I_{ab}$$

As a result, we have

$$I_{aa'} = \sqrt{3} I_p \angle 150^\circ$$

Similarly,

$$I_{bb'} = \sqrt{3} I_p \angle 30^\circ$$
$$I_{cc'} = \sqrt{3} I_p \angle -90^\circ$$

Note that a set of balanced three phase currents yields a corresponding set of balanced line currents that are $\sqrt{3}$ times the phase values:

$$I_L = \sqrt{3} I_p \tag{2.17}$$

where I_L denotes the magnitude of any of the three line currents.

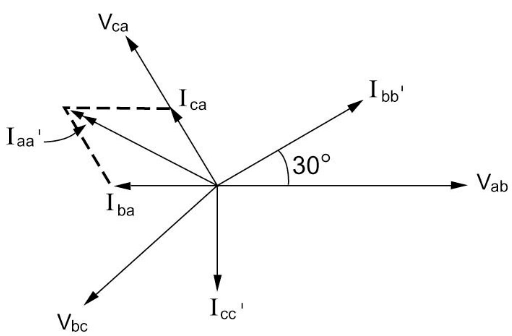


Figure 2.9 Illustrating Relation Between Phase and Line Currents in a Δ Connection.

Power Relationships

Assume that the three-phase generator is supplying a balanced load with the three sinusoidal phase voltages

$$\upsilon_a(t) = \sqrt{2}V_p \sin \omega t$$
$$\upsilon_b(t) = \sqrt{2}V_p \sin(\omega t - 120^\circ)$$
$$\upsilon_c(t) = \sqrt{2}V_p \sin(\omega t + 120^\circ)$$

With the currents given by

$$i_a(t) = \sqrt{2}I_p \sin(\omega t - \phi)$$
$$i_b(t) = \sqrt{2}I_p \sin(\omega t - 120^\circ - \phi)$$
$$i_c(t) = \sqrt{2}I_p \sin(\omega t + 120^\circ - \phi)$$

where ϕ is the phase angle between the current and voltage in each phase. The total power in the load is

$$p_{3\phi}(t) = \upsilon_a(t)i_a(t) + \upsilon_b(t)i_b(t) + \upsilon_c(t)i_c(t)$$

This turns out to be

$$p_{3\phi}(t) = V_p I_p \{3\cos\phi - [\cos(2\omega t - \phi) + \cos(2\omega t - 240 - \phi) \\ + \cos(2\omega t + 240 - \phi)]\}$$

Note that the last three terms in the above equation are the reactive power terms and they add up to zero. Thus we obtain

$$p_{3\phi}(t) = 3V_p I_p \cos\phi \tag{2.18}$$

The relationship between the line and phase voltages in a *Y*-connected system is

$$|V_L| = \sqrt{3}|V|$$

The power equation thus reads in terms of line quantities:

$$p_{3\phi} = \sqrt{3}|V_L||I_L|\cos\phi \tag{2.19}$$

The total instantaneous power is constant, having a magnitude of three times the real power per phase. We may be tempted to assume that the reactive power is of no importance in a three-phase system since the Q terms cancel out. However, this situation is analogous to the summation of balanced three-phase currents and voltages that also cancel out. Although the sum cancels out, these quantities are still very much in evidence in each phase. We thus extend the concept of complex or apparent power (S) to three-phase systems by defining

$$S_{3\phi} = 3V_p I_p^* \tag{2.20}$$

where the active power and reactive power are obtained from

$$S_{3\phi} = P_{3\phi} + jQ_{3\phi}$$

as

$$P_{3\phi} = 3|V_p||I_p|\cos\phi \tag{2.21}$$

$$Q_{3\phi} = 3|V_p||I_p|\sin\phi \tag{2.22}$$

and

$$P_{3\phi} = \sqrt{3}|V_L||I_L|\cos\phi \tag{2.23}$$

$$Q_{3\phi} = \sqrt{3}|V_L||I_L|\sin\phi \tag{2.24}$$

In specifying rated values for power system apparatus and equipment such as generators, transformers, circuit breakers, etc., we use he magnitude of the apparent power $S_{3\phi}$ as well as line voltage for specification values. In specifying three-phase motor loads, we use the horsepower output rating and voltage.

Example 2.2

A Y-connected, balanced three-phase load consisting of three impedances of $20\angle 30^\circ$ ohms each as shown in Figure 2.10 is supplied with the balanced line-to-neutral voltages:

$$V_{an} = 220\angle 0 \text{ V}$$
$$V_{bn} = 220\angle 240^\circ \text{ V}$$
$$V_{cn} = 220\angle 120^\circ \text{ V}$$

A. Calculate the phase currents in each line.
B. Calculate the line-to-line phasor voltages.
C. Calculate the total active and reactive power supplied to the load.

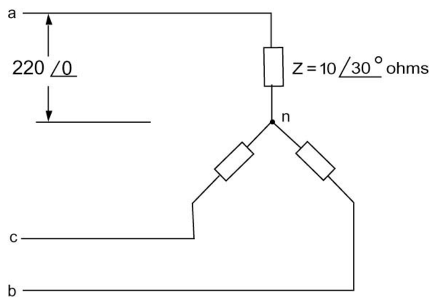


Figure 2.10 Load Connection for Example 2.2.

Solution

A. The phase currents are obtained as

$$I_{an} = \frac{220}{20\angle 30} = 11\angle -30^\circ \text{ A}$$
$$I_{bn} = \frac{220\angle 240}{20\angle 30} = 11\angle 210^\circ \text{ A}$$
$$I_{cn} = \frac{220\angle 120}{20\angle 30} = 11\angle 90^\circ \text{ A}$$

B. The line-to-line voltages are obtained as

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} \\ &= 220\angle 0 - 220\angle 240^\circ \\ &= 220\sqrt{3}\angle 30^\circ \\ V_{bc} &= 220\sqrt{3}\angle 30 - 120 = 220\sqrt{3}\angle -90^\circ \\ V_{ca} &= 220\sqrt{3}\angle -210^\circ \end{aligned}$$

C. The apparent power into phase *a* is given by

$$\begin{aligned} S_a &= V_{an} I_{an}^* \\ &= (220)(11)\angle 30^\circ \\ &= 2420\angle 30^\circ \text{ VA} \end{aligned}$$

The total apparent power is three times the phase value:

$$\begin{aligned} S_t &= 2420 \times 3\angle 30^\circ = 7260.00\angle 30^\circ \text{ VA} \\ &= 6287.35 + j3630.00 \end{aligned}$$

Thus

$$P_t = 6287.35 \text{ W}$$
$$Q_t = 3630.00 \text{ var}$$

Example 2.3

Repeat Example 2.2 as if the same three impedances were connected in a Δ connection.

Solution

From Example 2.2 we have

$$\begin{aligned} V_{ab} &= 220\sqrt{3}\angle 30^\circ \\ V_{bc} &= 220\sqrt{3}\angle -90^\circ \\ V_{ca} &= 220\sqrt{3}\angle -210^\circ \end{aligned}$$

The currents in each of the impedances are

$$\begin{aligned} I_{ab} &= \frac{220\sqrt{3}\angle 30^\circ}{20\angle 30} = 11\sqrt{3}\angle 0^\circ \\ I_{bc} &= 11\sqrt{3}\angle -120^\circ \\ I_{ca} &= 11\sqrt{3}\angle 120^\circ \end{aligned}$$

The line currents are obtained with reference to Figure 2.11 as

$$\begin{aligned} I_a &= I_{ab} - I_{ca} \\ &= 11\sqrt{3}\angle 0 - 11\sqrt{3}\angle -120^\circ \\ &= 33\angle 30^\circ \\ I_b &= I_{bc} - I_{ab} \\ &= 33\angle -90^\circ \\ I_c &= I_{ca} - I_{bc} \\ &= 33\angle -210^\circ \end{aligned}$$

The apparent power in the impedance between *a* and *b* is

$$\begin{aligned} S_{ab} &= V_{ab} I_{ab}^* \\ &= (220\sqrt{3}\angle 30^\circ)(22\sqrt{3}\angle 0) \\ &= 7260\angle 30^\circ \end{aligned}$$

The total three-phase power is then

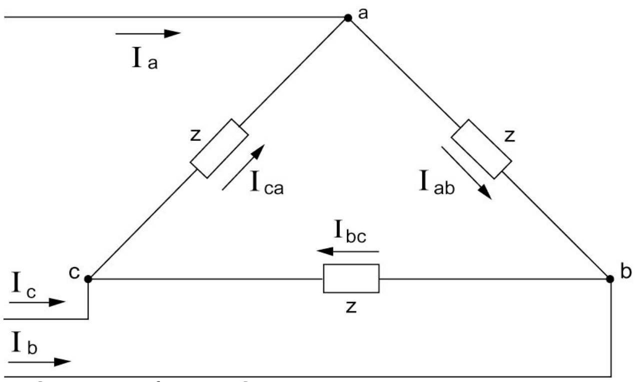


Figure 2.11 Load Connection for Example 2.3.

$$\begin{aligned} S_t &= 21780\angle 30^\circ \\ &= 18{,}862.02 + j10890.00 \end{aligned}$$

As a result,

$$\begin{aligned} P_t &= 37724.04 \text{ W} \\ Q_t &= 21780.00 \text{ var} \end{aligned}$$

2.4 THE PER UNIT SYSTEM

The per unit (p.u.) value representation of electrical variables in power system problems is favored in electric power systems. The numerical per unit value of any quantity is its ratio to a chosen base quantity of the same dimension. Thus a per unit quantity is a *normalized* quantity with respect to the chosen base value. The per unit value of a quantity is thus defined as

$$\text{p.u. value} = \frac{\text{Actual value}}{\text{Reference or base value of hte same dimension}} \tag{2.25}$$

Five quantities are involved in the calculations. These are the current *I*, the voltage *V*, the complex power *S*, the impedance *Z*, and the phase angles. The angles are dimensionless; the other four quantities are completely described by knowledge of only two of them. An arbitrary choice of two base quantities will fix the other base quantities. Let $|I_b|$ and $|V_b|$ represent the base current and base voltage expressed in kiloamperes and kilovolts, respectively. The product of the two gives the base complex power in megavoltamperes (MVA)

$$|S_b| = |V_b||I_b| \text{MVA} \tag{2.26}$$

The base impedance will also be given by

$$|Z_b| = \frac{|V_b|}{|I_b|} = \frac{|V_b|^2}{|S_b|} \text{ ohms} \tag{2.27}$$

The base admittance will naturally be the inverse of the base impedance. Thus,

$$\begin{aligned} |Y_b| &= \frac{1}{|Z_b|} = \frac{|I_b|}{|V_b|} \\ &= \frac{|S_b|}{|V_b|^2} \text{ siemens} \end{aligned} \tag{2.28}$$

The nominal voltage of lines and equipment is almost always known as well as the apparent (complex) power in megavoltamperes, so these two quantities are usually chosen for base value calculation. The same megavoltampere base is used in all parts of a given system. One base voltage is chosen; all other base voltages must then be related to the one chosen by the turns ratios of the connecting transformers.

From the definition of per unit impedance, we can express the ohmic impedance Z_Ω in the per unit value $Z_{\text{p.u.}}$ as

$$Z_{\text{p.u.}} = \frac{Z_\Omega |S_b|}{|V_b|^2} \text{ p.u.} \tag{2.29}$$

As for admittances, we have

$$Y_{\text{p.u.}} \overset{\Delta}{=} \frac{1}{Z_{\text{p.u.}}} = \frac{|V_b|^2}{Z_\Omega |S_b|} = Y_S \frac{|V_b|^2}{|S_b|} \text{ p.u.} \tag{2.30}$$

Note that $Z_{\text{p.u.}}$ can be interpreted as the ratio of the voltage drop across Z with base current injected to the base voltage.

Example 2.4

Consider a transmission line with $Z = 3.346 + j77.299\Omega$. Assume that

$$\begin{aligned} S_b &= 100 \text{ MVA} \\ V_b &= 735 \text{ kV} \end{aligned}$$

We thus have

$$Z_{\text{p.u.}} = Z_\Omega \cdot \frac{S_b}{|V_b|^2} = Z_\Omega \cdot \frac{1000}{(735)^2}$$
$$= 1.85108 \times 10^{-4} (Z_\Omega)$$

For R = 3.346 ohms we obtain

$$R_{\text{p.u.}} = (3.346)\left(1.85108 \times 10^{-4}\right) = 6.19372 \times 10^{-4}$$

For X = 77.299 ohms, we obtain

$$X_{\text{p.u.}} = (77.299)\left(1.85108 \times 10^{-4}\right) = 1.430867 \times 10^{-2}$$

For the admittance we have

$$Y_{\text{p.u.}} = Y_S \cdot \frac{|V_b|^2}{S_b}$$
$$= Y_S \frac{(735)^2}{100}$$
$$= 5.40225 \times 10^3 (Y_S)$$

For $Y = 1.106065 \times 10^{-3}$ siemens, we obtain

$$Y_{\text{p.u.}} = \left(5.40225 \times 10^3\right)\left(1.106065 \times 10^{-3}\right)$$
$$= 5.97524$$

Base Conversions

Given an impedance in per unit on a given base S_{b_0} and V_{b_0}, it is sometimes required to obtain the per unit value referred to a new base set S_{b_n} and V_{b_n}. The conversion expression is obtained as:

$$Z_{\text{p.u.}_n} = Z_{\text{p.u.}_0} \frac{|S_{b_n}|}{|S_{b_0}|} \cdot \frac{|V_{b_0}|^2}{|V_{b_n}|^2} \qquad \textbf{(2.31)}$$

which is our required conversion formula. The admittance case simply follows the inverse rule. Thus,

$$Y_{\text{p.u.}_n} = Y_{\text{p.u.}_0} \frac{\left|S_{b_0}\right|}{\left|S_{b_n}\right|} \cdot \frac{\left|V_{b_n}\right|^2}{\left|V_{b_0}\right|^2} \tag{2.32}$$

Example 2.5

Convert the impedance and admittance values of Example 2.4 to the new base of 200 MVA and 345 kV.

Solution

We have

$$Z_{\text{p.u.}_0} = 6.19372\times10^{-4} + j1.430867\times10^{-2}$$

for a 100-MVA, 735-kV base. With a new base of 200 MVA and 345 kV, we have, using the impedance conversion formula,

$$\begin{aligned} Z_{\text{p.u.}_n} &= Z_{\text{p.u.}_0}\left(\frac{200}{100}\right)\cdot\left(\frac{735}{345}\right)^2 \\ &= 9.0775 Z_{\text{p.u.}_0} \end{aligned}$$

Thus,

$$Z_{\text{p.u.}_n} = 5.6224\times10^{-3} + j1.2989\times10^{-1}\ \text{p.u.}$$

For the admittance we have

$$\begin{aligned} Y_{\text{p.u.}_n} &= Y_{\text{p.u.}_0}\left(\frac{100}{200}\right)\cdot\left(\frac{345}{735}\right)^2 \\ &= 0.11016 Y_{\text{p.u.}_0} \end{aligned}$$

Thus,

$$\begin{aligned} Y_{\text{p.u.}_n} &= (5.97524)(0.11016) \\ &= 0.65825\ \text{p.u.} \end{aligned}$$

2.5 ELECTROMAGNETISM AND ELECTROMECHANICAL ENERGY CONVERSION

An electromechanical energy conversion device transfers energy between an input side and an output side, as shown in Figure 2.12. In an electric motor, the input is electrical energy drawn from the supply source and the output is mechanical energy supplied to the load, which may be a pump, fan, hoist, or any other mechanical load. An electric generator converts mechanical energy

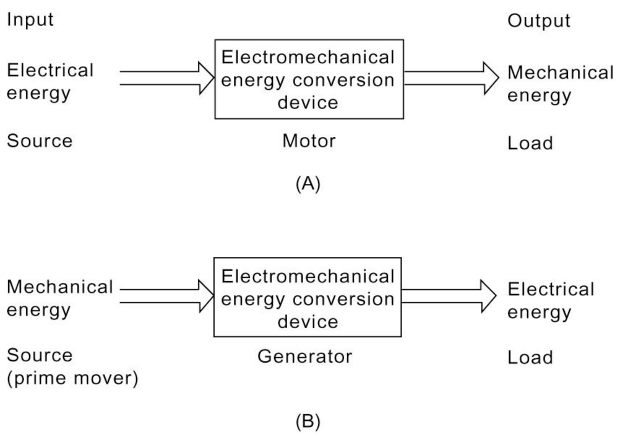


Figure 2.12 Functional block diagram of electromechanical energy conversion devices as (A) motor, and (B) generator.

supplied by a prime mover to electrical form at the output side. The operation of electromechanical energy conversion devices is based on fundamental principles resulting from experimental work.

Stationary electric charges produce electric fields. On the other hand, magnetic field is associated with moving charges and thus electric currents are sources of magnetic fields. A magnetic field is identified by a vector **B** called the magnetic flux density. In the SI system of units, the unit of **B** is the tesla (T). The magnetic flux $\Phi = \mathbf{B}.\mathbf{A}$. The unit of magnetic flux Φ in the SI system of units is the weber (Wb).

The Lorentz Force Law

A charged particle q, in motion at a velocity **V** in a magnetic field of flux density **B**, is found experimentally to experience a force whose magnitude is proportional to the product of the magnitude of the charge q, its velocity, and the flux density **B** and to the sine of the angle between the vectors **V** and **B** and is given by a vector in the direction of the cross product $\mathbf{V} \times \mathbf{B}$. Thus we write

$$\mathbf{F} = q\mathbf{V} \times \mathbf{B} \tag{2.33}$$

Equation (2.33) is known as the Lorentz force equation. The direction of the force is perpendicular to the plane of **V** and **B** and follows the right-hand rule. An interpretation of Eq. (2.33) is given in Figure 2.13.

The tesla can then be defined as the magnetic flux density that exists when a charge q of 1 coulomb, moving normal to the field at a velocity of 1 m/s, experiences a force of 1 newton.

A distribution of charge experiences a differential force $d\mathbf{F}$ on each moving incremental charge element dq given by

$$d\mathbf{F} = dq\,(\mathbf{V} \times \mathbf{B})$$

Moving charges over a line constitute a line current and thus we have

$$d\mathbf{F} = (I \times \mathbf{B})\,\mathbf{dl} \tag{2.34}$$

Equation (2.34) simply states that a current element I **dl** in a magnetic field **B** will experience a force d**F** given by the cross product of I **dl** and **B**. A pictorial presentation of Eq. (2.34) is given in Figure 2.14.

The current element I **dl** cannot exist by itself and must be a part of a complete circuit. The force on an entire loop can be obtained by integrating the current element

$$\mathbf{F} = \oint I\,\mathbf{dl} \times \mathbf{B} \tag{2.35}$$

Equations (2.34) and (2.35) are fundamental in the analysis and design of electric motors, as will be seen later.

The Biot-Savart law is based on Ampère's work showing that electric currents exert forces on each other and that a magnet could be replaced by an equivalent current.

Consider a long straight wire carrying a current I as shown in Figure 2.15. Application of the Biot-Savart law allows us to find the total field at P as:

$$\mathrm{B} = \frac{\mu_0 I}{2\pi R} \tag{2.36}$$

The constant μ_0 is called the permeability of free space and in SI units is given by

$$\mu_0 = 4\pi \times 10^{-7}$$

The magnetic field is in the form of concentric circles about the wire,

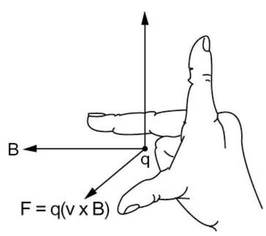


Figure 2.13 Lorentz force law.

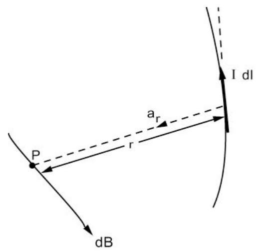


Figure 2.14 Interpreting the Biot-Savart law.

with a magnitude that increases in proportion to the current I and decreases as the distance from the wire is increased.

The Biot-Savart law provides us with a relation between current and the resulting magnetic flux density **B**. An alternative to this relation is Ampère's circuital law, which states that the line integral of **B** about any closed path in free space is exactly equal to the current enclosed by that path times μ_0.

$$\oint_c \mathbf{B} \cdot \mathbf{dl} = \begin{cases} \mu_0 I & \text{path } c \text{ encloses } I \\ 0 & \text{path } c \text{ does not enclose } I \end{cases} \qquad \textbf{(2.37)}$$

It should be noted that the path c can be arbitrarily shaped closed loop about the net current I.

2.6 PERMEABILITY AND MAGNETIC FIELD INTENSITY

To extend magnetic field laws to materials that exhibit a linear variation of **B** with I, all expressions are valid provided that μ_0 is replaced by the permeability corresponding to the material considered. From a **B**-I – variation point of view we divide materials into two classes:

1. Nonmagnetic material such as all dielectrics and metals with permeability equal to μ_0 for all practical purposes.
2. Magnetic material such as ferromagnetic material (the iron group), where a given current produces a much larger **B** field than in free space. The permeability in this case is much higher than that of free space and varies with current in a nonlinear manner over a wide range. Ferromagnetic material can be further categorized into two classes:

   a) Soft ferromagnetic material for which a linearization of the **B**-I variation in a region is possible. The source of **B** in the case of soft ferromagnetic material can be modeled as due to the current I.
   b) Hard ferromagnetic material for which it is difficult to

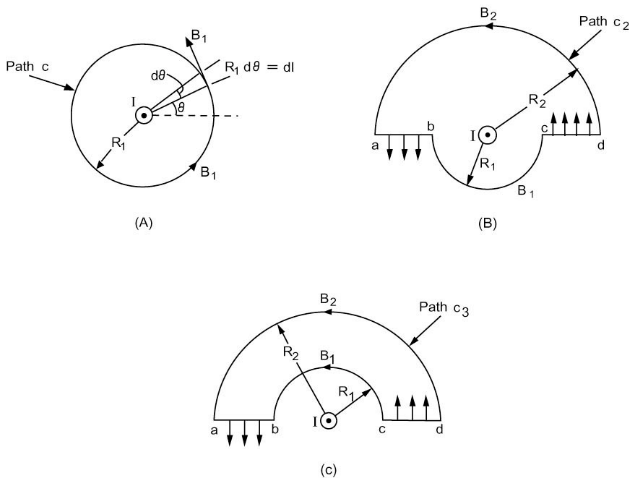


Figure 2.15 Illustrating Ampère's circuital law: (A) path c_1 is a circle enclosing current *I*, (B) path c_2 is not a circle but encloses current *I*, and (C) path c_3 does not enclose current *I*.

give a meaning to the term *permeability*. Material in this group is suitable for permanent magnets.

For hard ferromagnetic material, the source of **B** is a combined effect of current *I* and material magnetization *M*, which originates entirely in the medium. To separate the two sources of the magnetic **B** field, the concept of magnetic field intensity **H** is introduced.

Magnetic Field Intensity

The magnetic field intensity (or strength) denoted by **H** is a vector defined by the relation

$$\mathbf{B} = \mu \mathbf{H} \tag{2.38}$$

For isotropic media (having the same properties in all directions), μ is a scalar and thus **B** and **H** are in the same direction. On the basis of Eq. (2.38), we can write the statement of Ampère's circuital law as

$$\int \mathbf{H} \cdot \mathbf{dl} = \begin{cases} I & \text{path } c \text{ encloses } I \\ 0 & \text{path } c \text{ does not enclose } I \end{cases} \tag{2.39}$$

The expression in Eq. (2.39) is independent of the medium and relates the magnetic field intensity **H** to the current causing it, *I*.

Permeability μ is not a constant in general but is dependent on **H** and, strictly speaking, one should state this dependence in the form

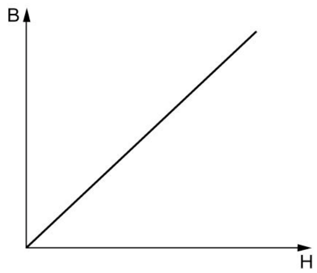


Figure 2.16 B-H characteristic for nonmagnetic material.

$$\mu = \mu(\mathbf{H}) \tag{2.40}$$

For nonmagnetic material, μ is constant at a value equal to $\mu_0 = 4\pi \times 10^{-7}$ for all practical purposes. The **B**-**H** characteristic of nonmagnetic materials is shown in Figure 2.14

The **B**-**H** characteristics of soft ferromagnetic material, often called the magnetization curve, follow the typical pattern displayed in Figure 2.15. The permeability of the material in accordance with Eq. (2.38) is given as the ratio of **B** to **H** and is clearly a function of **H**, as indicated by Eq. (2.40).

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} \tag{2.41}$$

The permeability at low values of **H** is called the initial permeability and is much lower than the permeability at higher values of **H**. The maximum value of μ occurs at the knee of the **B**-**H** characteristic. The permeability of soft ferromagnetic material μ is much larger than μ_0 and it is convenient to define the relative permeability μ_r by

$$\mu_r = \frac{\mu}{\mu_0} \tag{2.42}$$

A typical variation of μ_r with **H** for a ferromagnetic material is shown in Figure 2.18.

For practical electromechanical energy conversion devices, a linear approximation to the magnetization curve provides satisfactory answers in the normal region of operation. The main idea is to fit a straight line passing through the origin of the **B**-**H** curve that best fits the data points which is drawn and taken to represent the characteristics of the material considered. Within the acceptable range of **H** values, one may then use the following relation to model the ferromagnetic material:

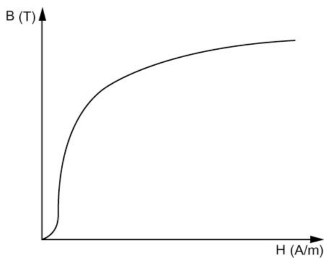


Figure 2.17 B-H characteristic for a typical ferromagnetic material.

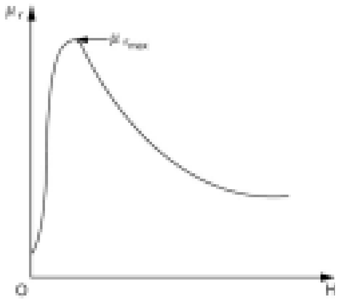


Figure 2.18 Typical variation of μ_r with H for a ferromagnetic material.

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} \tag{2.43}$$

It should be noted that μ_r is in the order of thousands for magnetic materials used in electromechanical energy conversion devices (2000 to 80,000, typically). Properties of magnetic materials are discussed further in the following sections. Presently, we assume that μ_r is constant.

2.7 FLUX LINKAGES, INDUCED VOLTAGES, INDUCTANCE, AND ENERGY

A change in a magnetic field establishes an electric field that is manifested as an induced voltage. This basic fact is due to Faraday's experiments and is expressed by Faraday's law of electromagnetic induction.

Consider a toroidal coil with N turns through which a current i flows producing a total flux Φ. Each turn encloses or links the total flux and we also note that the total flux links each of the N turns. In this situation, we define the flux linkages λ as the product of the number of turns N and the flux Φ linking each turn.

$$\lambda = N\Phi \tag{2.44}$$

The flux linkages λ can be related to the current i in the coil by the definition of inductance L through the relation

$$\lambda = Li \tag{2.45}$$

Inductance is the passive circuit element that is related to the geometry and material properties of the structure. From this point of view, inductance is the ratio of total flux linkages to the current, which the flux links. The inductance L is related to the reluctance $\Re$ of the magnetic structure of a single-loop structure.

$$L = \frac{N^2}{\Re} \tag{2.46}$$

In the case of a toroid with a linear **B**-**H** curve, we have

$$L = \frac{N^2 A}{l}\mu_0\mu_r \tag{2.47}$$

There is no single definition of inductance which is useful in all cases for which the medium is not linear. The unit of inductance is the henry or weber-turns per ampere.

In terms of flux linkages, Faraday's law is stated as

$$e = \frac{d\lambda}{dt} = N\frac{d\Phi}{dt} \tag{2.48}$$

The electromotive force (EMF), or induce voltage, is thus equal to the rate of change of flux linkages in the structure. We also write:

$$e = \frac{d}{dt}Li \tag{2.49}$$

In electromechanical energy conversion devices, the reluctance varies with time and thus L also varies with time. In this case,

$$e = L\frac{di}{dt} + i\frac{dL}{dt} \tag{2.50}$$

Note that if L is constant, we get the familiar equation for modeling an inductor in elementary circuit analysis.

Power and energy relationships in a magnetic circuit are important in evaluating performance of electromechanical energy conversion devices treated in this book. We presently explore some basic relationships, starting with the fundamental definition of power *p(t)* given by

$$p(t) = e(t)i(t) \qquad \textbf{(2.51)}$$

The power into a component (the coil in the case of toroid) is given as the product of the voltage across its terminals *e(t)* and the current through *i(t)*. Using Faraday's law, see Eq. (2.50), we can write

$$p(t) = i(t)\frac{d\lambda}{dt} \qquad \textbf{(2.52)}$$

The units of power are watts (or joules per second).

Let us recall the basic relation stating that power *p(t)* is the rate of change of energy *W(t)*:

$$p(t) = \frac{dW}{dt} \qquad \textbf{(2.53)}$$

We can show that

$$dW = (lA)H\ dB \qquad \textbf{(2.54)}$$

Consider the case of a magnetic structure that experiences a change in state between the time instants t_1 and t_2. Then, change in energy into the system is denoted by ΔW and is given by

$$\Delta W = W(t_2) - W(t_1) \qquad \textbf{(2.55)}$$

We can show that

$$\Delta W = lA\int_{\mathbf{B}_1}^{\mathbf{B}_2} \mathrm{H}d\mathrm{B} \qquad \textbf{(2.56)}$$

It is clear that the energy per unit volume expended between t_1 and t_2 is the area between the **B**-**H** curve and the B axis between B_1 and B_2.

It is important to realize that the energy relations obtained thus far do not require linearity of the characteristics. For a linear structure, we can develop these relations further. We can show that

$$\Delta W = \frac{1}{2L}(\lambda_2^2 - \lambda_1^2) \qquad \textbf{(2.57)}$$

or

$$\Delta W = \tfrac{1}{2}L(i_2^2 - i_1^2) \qquad \textbf{(2.58)}$$

The energy expressions obtained in this section provide us with measures of energy stored in the magnetic field treated. This information is useful in many ways, as will be seen in this text.

2.8 HYSTERESIS LOOP

Ferromagnetic materials are characterized by a **B**-**H** characteristic that is both nonlinear and multivalued. This is generally referred to as a hysteresis characteristic. To illustrate this phenomenon, we use the sequence of portraits of Figure 2.19 showing the evolution of a hysteresis loop for a toroid with virgin ferromagnetic core. Assume that the MMF (and hence **H**) is a slowly varying sinusoidal waveform with period T as shown in the lower portion graphs of Figure 2.19. We will discuss the evolution of the **B**-**H** hysteresis loop in the following intervals.

> *Interval I*: Between $t = 0$ and $T/4$, the magnetic field intensity **H** is positive and increasing. The flux density increases along the initial curve (*oa*) up to the saturation value $\mathbf{B}_s$. Increasing **H** beyond saturation level does not result in an increase in **B**.
> *Interval II*: Between $t = T/4$ and $T/2$, the magnetic field intensity is positive but decreasing. The flux density **B** is observed to decrease along the segment *ab*. Note that *ab* is above *oa* and thus for the same value of **H**, we get a different value of **B**. This is true at *b*, where there is a value for $\mathbf{B} = \mathbf{B}_r$ different from zero even though **H** is zero at that point in time $t = T/2$. The value of $\mathbf{B}_r$ is referred to as the residual field, remanence, or retentivity. If we leave the coil unenergized, the core will still be magnetized.
> *Interval III*: Between $t = T/2$ and $3T/4$, the magnetic field intensity **H** is reversed and increases in magnitude. **B** decreases to zero at point *c*. The value of **H**, at which magnetization is zero, is called the coercive force $\mathbf{H}_c$. Further decrease in **H** results in reversal of **B** up to point *d*, corresponding to $t = 3T/4$.
> *Interval IV*: Between $t = 2T/4$ and T, the value of **H** is negative but increasing. The flux density **B** is negative and increases from *d* to *e*. Residual field is observed at *e* with $\mathbf{H} = 0$.
> *Interval V*: Between $t = T$ and $5T/4$, **H** is increased from 0, and the flux density is negative but increasing up to *f*, where the material is demagnetized. Beyond *f*, we find that **B** increases up to *a* again.

A typical hysteresis loop is shown in Figure 2.20. On the same graph, the **B**-**H** characteristic for nonmagnetic material is shown to show the relative magnitudes involved. It should be noted that for each maximum value of the ac magnetic field intensity cycle, there is a steady-state loop, as shown in Figure 2.21. The dashed curve connecting the tips of the loops in the figure is the dc magnetization curve for the material. Table 2.1 lists some typical values for $\mathbf{H}_c$, $\mathbf{B}_r$, and $\mathbf{B}_s$ for common magnetic materials.

We know that the energy supplied by the source per unit volume of the magnetic structure is given by

$$d\tilde{W} = \mathrm{H}d\mathrm{B}$$

and

$$\Delta\tilde{W} = \int_{\mathbf{B}_1}^{\mathbf{B}} \mathrm{H}d\mathrm{B}$$

The energy supplied by the source in moving from a to b in the graph of Figure 2.22(A) is negative since **H** is positive but **B** is decreasing. If we continue on from *b* to *d* through *c*, the energy is positive as **H** is negative but **B** is decreasing.

The second half of the loop is treated in Figure 2.22(B) and is self-explanatory. Superimposing both halves of the loop, we obtain Figure 2.22(C), which clearly shows that the net energy per unit volume supplied by the source is the area enclosed by the hysteresis loop. This energy is expended in the magnetization-demagnetization process and is dissipated as a heat loss. Note that the loop is described in one cycle and as a result, the hysteresis loss per second is equal to the product of the loop area and the frequency *f* of the waveform applied. The area of the loop depends on the maximum flux density, and as a result, we assert that the power dissipated through hysteresis P_h is given by

$$P_h = k_h f(\mathrm{B}_m)^n$$

Where k_h is a constant, *f* is the frequency, and B_m is the maximum flux density. The exponent *n* is determined from experimental results and ranges between 1.5 and 2.5.

2.9 EDDY CURRENT AND CORE LOSSES

If the core is subject to a time-varying magnetic field (sinusoidal input was assumed), energy is extracted from the source in the form of hysteresis losses. There is another loss mechanism that arises in connection with the application of time-varying magnetic field, called eddy-current loss. A rigorous analysis of the eddy-current phenomenon is a complex process but the basic model can be explained in simple terms on the basis of Faraday’s law.

The change in flux will induce voltages in the core material which will result in currents circulating in the core. The induced currents tend to establish a flux that opposes the original change imposed by the source. The induced currents, which are essentially the eddy currents, will result in power loss due to heating of the core material. To minimize eddy current losses, the magnetic core is made of stackings of sheet steel laminations, ideally separated by highly resistive material. It is clear that this effectively results in the actual area of the

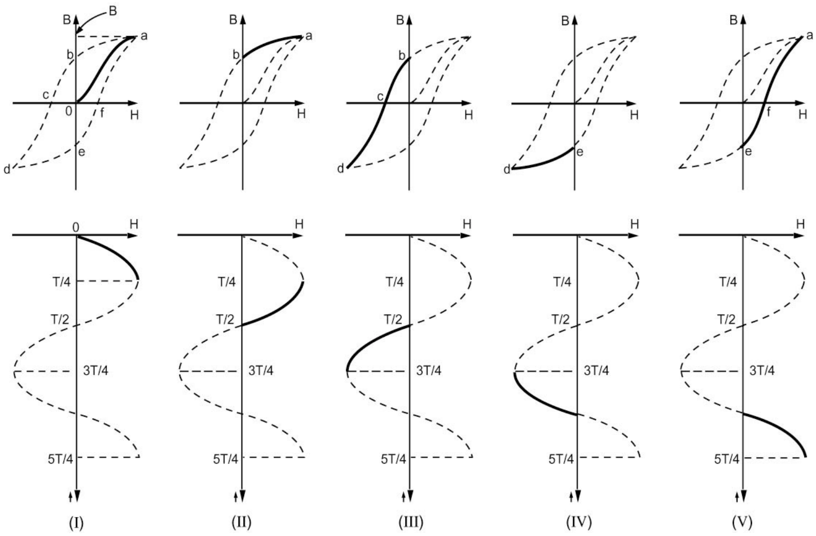


Figure 2.19 Evolution of the hysteresis loop.

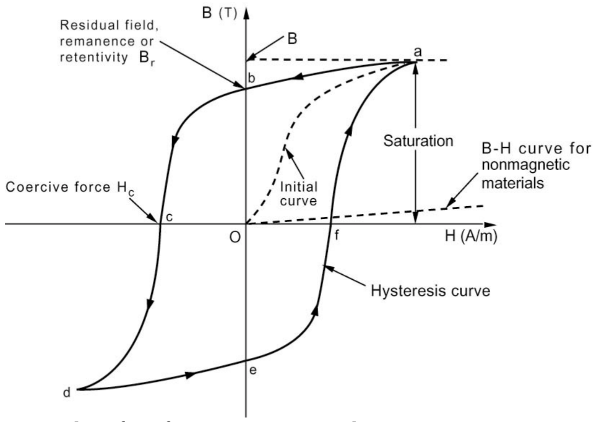


Figure 2.20 Hysteresis loop for a ferromagnetic material.

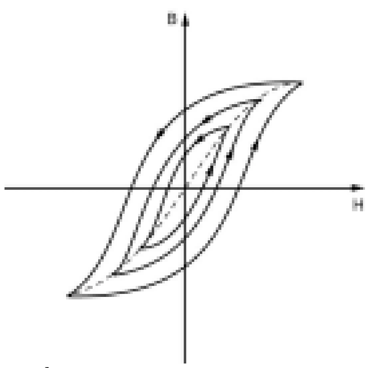


Figure 2.21 Family of hysteresis loops.

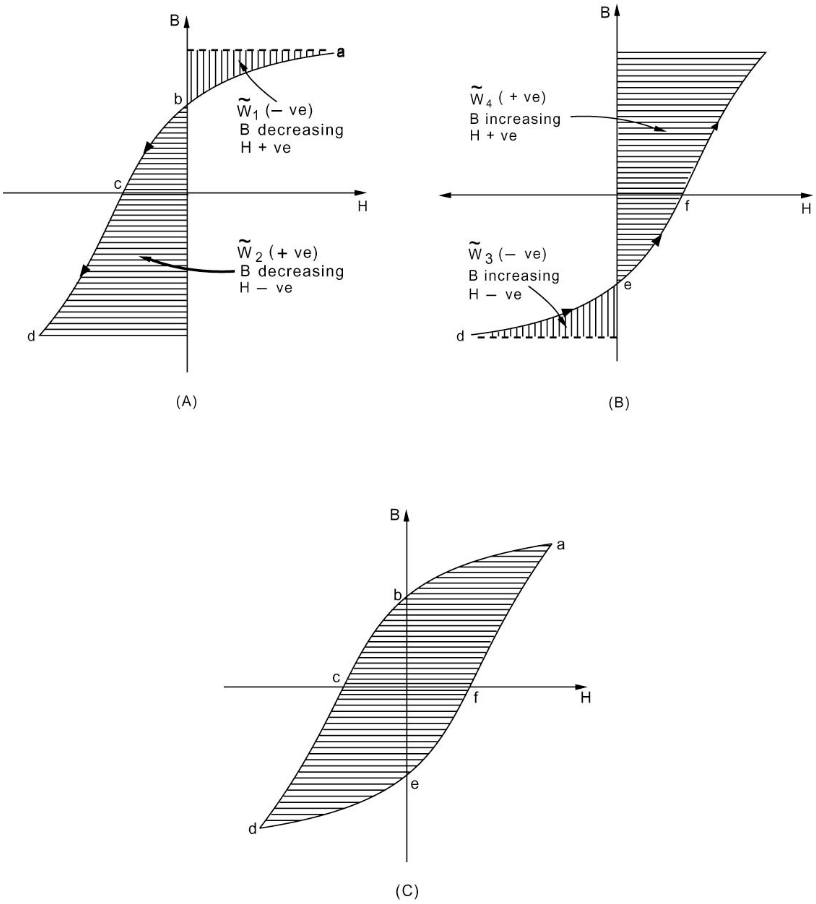


Figure 2.22 Illustrating the concept of energy loss in the hysteresis process.

Table 2.1
Properties of Magnetic Materials and Magnetic Alloys

Material (Composition)	*Initial Relative Permeability,* μ_i/μ_0	*Maximum Relative Permeability,* μ_{max}/μ_0	*Coercive Force* H_r (A/m)	*Residual Field* B_r (Wb/m^2)	*Saturation Field* B_s (Wb/m^2)
Commercial iron (0.2 imp)	250	9,000	$\cong 80$	0.77	2.15
Silicon-iron (4 Si)	1,500	7,000	20	0.5	1.95
Silicon-iron (3 Si)	7,500	55,000	8	0.95	2.00
Mu metal (5 Cu, 2 Cr, 77 Ni)	20,000	100,000	4	0.23	0.65
78 Permalloy (78.5 Ni)	8,000	100,000	4	0.6	1.08
Supermalloy (79 Ni, 5 Mo)	100,000	1,000,000	0.16	0.5	0.79

magnetic material being less than the gross area presented by the stack. To account for this, a stacking factor is employed for practical circuit calculations.

$$\text{Stacking factor} = \frac{\text{actual magnetic cross-sectional area}}{\text{gross cross-sectional area}}$$

Typically, lamination thickness ranges from 0.01 mm to 0.35 m with associated stacking factors ranging between 0.5 to 0.95. The eddy-current power loss per unit volume can be expressed by the empirical formula

$$P_c = K_e(f\mathrm{B}_m t_1)^2 \qquad \mathrm{W/m^3}$$

The eddy-current power loss per unit volume varies with the square of frequency f, maximum flux density B_m, and the lamination thickness t_1. K_e is a proportionality constant.

The term *core loss* is used to denote the combination of eddy-current and hysteresis power losses in the material. In practice, manufacturer-supplied data are used to estimate the core loss P_c for given frequencies and flux densities for a particular type of material.

2.10 ENERGY FLOW APPROACH

From an energy flow point of view consider an electromechanical energy conversion device operating as a motor. We develop a model of the process that is practical and easy to follow and therefore take a macroscopic approach based on the principle of energy conservation. The situation is best illustrated using the diagrams of Figure 2.23. We assume that an incremental change in electric energy supply dW_e has taken place. This energy flow into the device can be visualized as being made up of three components, as shown in Figure 2.23(A). Part of the energy will be imparted to the magnetic field of the device and will result in an increase in the energy stored in the field, denoted by dW_f. A second component of energy will be expended as heat losses dW_{loss}. The third and most important component is that output energy be made available to the load (dW_{mech}). The heat losses are due to ohmic (I^2R) losses in the stator (stationary member) and rotor (rotating member); iron or field losses through eddy current and hysteresis as discussed earlier; and mechanical losses in the form of friction and windage.

In part (B) of Figure 2.23, the energy flow is shown in a form that is closer to reality by visualizing Ampère's bonne homme making a trip through the machine. Starting in the stator, ohmic losses will be encountered, followed by field losses and a change in the energy stored in the magnetic field. Having crossed the air gap, our friend will witness ohmic losses in the rotor windings taking place, and in passing to the shaft, bearing frictional losses are also encountered. Finally, a mechanical energy output is available to the load. It should be emphasized here that the phenomena dealt with are distributed in nature and what we are doing is simply developing an understanding in the form of mathematical expressions called models. The trip by our Amperean friend can never take place in real life but is a helpful means of visualizing the process.

We write the energy balance equation based on the previous arguments. Here we write

$$dW_e = dW_{\text{fld}} + dW_{\text{loss}} + dW_{\text{mech}} \tag{2.59}$$

To simplify the treatment, let us assume that losses are negligible.

The electric power input $P_e(t)$ to the device is given in terms of the terminal voltage $e(t)$ and current $i(t)$, and using it, we write Faraday's law:

$$P_e(t)dt = i(t)d\lambda \tag{2.60}$$

We recognize the left-hand side of the equation as being the increment in electric energy dW_e, and we therefore write

$$dW_e = id\lambda \tag{2.61}$$

Assuming a lossless device, we can therefore write an energy balance equation which is a modification of Eq. (2.61).

$$dW_e = dW_{\text{fld}} + dW_{\text{mech}} \tag{2.62}$$

The increment in mechanical output energy can be expressed in the case of a translational (linear motion) increment dx and the associated force exerted by the field F_{fld} as

$$dW_{\text{mech}} = F_{\text{fld}}dx \tag{2.63}$$

In the case of rotary motion, the force is replaced by torque T_{fld} and the linear increment dx is replaced by the angular increment $d\theta$:

$$dW_{\text{mech}} = T_{\text{fld}}d\theta \tag{2.64}$$

As a result, we have for the case of linear motion,

$$dW_{\text{fld}} = id\lambda - F_{\text{fld}}dx \tag{2.65}$$

And for rotary motion,

$$dW_{\text{fld}} = id\lambda - T_{\text{fld}}d\theta \tag{2.66}$$

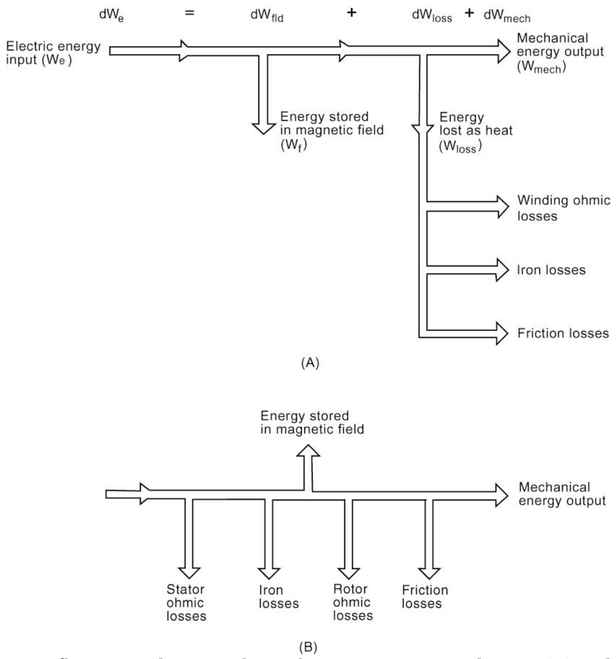


Figure 2.23 Energy flow in an electromechanical energy conversion device: (A) with losses segregated, and (B) more realistic representation.

The foregoing results state that the net change in the field energy is obtained through knowledge of the incremental electric energy input ($i\ d\lambda$) and the mechanical increment of work done.

The field energy is a function of two states of the system. The first is the displacement variable x (or θ for rotary motion), and the second is either the flux linkages λ or the current i. This follows since knowledge of λ completely specifies i through the λ-i characteristic. Let us first take dependence of W_f on λ and x, and write

$$dW_{\text{fld}}(\lambda, x) = \frac{\partial W_f}{\partial \lambda} d\lambda + \frac{\partial W_f}{\partial x} dx \tag{2.67}$$

The incremental increase in field energy W_f is made up of two components. The first is the product of $d\lambda$ and a (gain factor) coefficient equal to the partial derivative of W_f with respect to λ (x is held constant); the second component is equal to the product of dx and the partial derivative of W_f with respect to x (λ is held constant). This is a consequence of Taylor's series for a function of two variables. We conclude that

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \tag{2.68}$$

$$F_{\text{fld}} = -\frac{\partial W_f(\lambda, x)}{\partial x} \tag{2.69}$$

This result states that if the energy stored in the field is known as a function of λ and x, then the electric force developed can be obtained by the partial differentiation shown in Eq. (2.69).

For rotary motion, we replace x by θ in the foregoing development to arrive at

$$T_{\text{fld}} = \frac{-\partial W_f(\lambda, x)}{\partial \theta} \tag{2.70}$$

Of course, W_f as a function of λ and θ must be available to obtain the developed torque. Our next task, therefore, is to determine the variations of the field energy with λ and x for linear motion and that with λ and θ for rotary motion.

Field Energy

To find the field force we need an expression for the field energy $W_f(\lambda_p, x_p)$ at a given state λ_p and x_p. This can be obtained by integrating the relation of Eq. (2.70) to obtain

$$W_f(\lambda_p, x_p) = \int_0^{\lambda p} i(\lambda_p, x_p)\, d\lambda \tag{2.71}$$

If the λ–i characteristic is linear in i then

$$W_f(\lambda_p, x_p) = \frac{\lambda_p^2}{2L} \tag{2.72}$$

Note that L can be a function of x.

Coil Voltage

Using Faraday's law, we have

$$e(t) = \frac{d\lambda}{dt} = \frac{d}{dt}(Li)$$

Thus, since L is time dependent, we have

$$e(t) = L\frac{di}{dt} + i\frac{dL}{dt}$$

However,

$$\frac{dL}{dt}=\frac{dL}{dx}\left(\frac{dx}{d}\right)=\upsilon\frac{dL}{dx}$$

As a result, we assert that the coil voltage is given by

$$e(t)=L\frac{di}{dt}+i\upsilon\frac{dL}{dx} \quad \textbf{(2.73)}$$

where $\upsilon = dx/dt$.

2.11 MULTIPLY EXCITED SYSTEMS

Most rotating electromechanical energy conversion devices have more than one exciting winding and are referred to as multiply excited systems. The torque produced can be obtained by a simple extension of the techniques discussed earlier. Consider a system with three windings as shown in Figure 2.24.

The differential electric energy input is

$$dW_e = i_1 d\lambda_1 + i_2 d\lambda_2 + i_3 d\lambda_3 \quad \textbf{(2.74)}$$

The mechanical energy increment is given by

$$dW_{\text{mech}} = T_{\text{fld}}\, d\theta$$

Thus, the field energy increment is obtained as

$$\begin{aligned} dW_{\text{fld}} &= dW_e - dW_{\text{mech}} \\ &= i_1 d\lambda_1 + i_2 d\lambda_2 + i_3 d\lambda_3 - T_{\text{fld}}\, d\theta \end{aligned} \quad \textbf{(2.75)}$$

If we express W_{fld} in terms of λ_1, λ_2, λ_3, and θ, we have

$$dW_f = \frac{\partial W_f}{\partial \lambda_1}d\lambda_1 + \frac{\partial W_f}{\partial \lambda_2}d\lambda_2 + \frac{\partial W_f}{\partial \lambda_3}d\lambda_3 + \frac{\partial W_f}{\partial \theta}d\theta \quad \textbf{(2.76)}$$

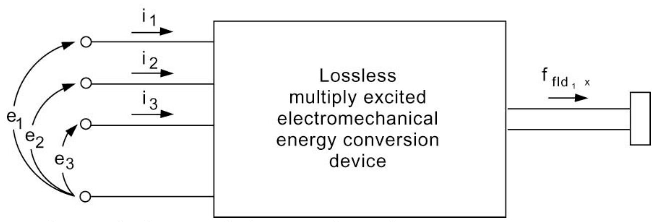


Figure 2.24 Lossless Multiply Excited Electromechanical Energy Conversion Device.

By comparing Eqs. (2.75) and (2.76), we conclude:

$$T_{\text{fld}} = -\frac{\partial W_f(\lambda_1, \lambda_2, \lambda_3, \theta)}{\partial \theta} \tag{2.77}$$

$$i_k = \frac{\partial W_f(\lambda_1, \lambda_2, \lambda_3, \theta)}{\partial \lambda_k} \tag{2.78}$$

The field energy at a state corresponding to point P, where $\lambda_1 = \lambda_{1p}$, $\lambda_2 = \lambda_{2p}$, $\lambda_3 = \lambda_{3p}$, and $\theta = \theta_p$ is obtained as:

$$W_f(\lambda_{1p}, \lambda_{2p}, \lambda_{3p}, \theta_p) = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{1}{2} \lambda_{ip} \Gamma_{ij} \lambda_{jp} \tag{2.79}$$

where

$$i_1 = \Gamma_{11}\lambda_1 + \Gamma_{12}\lambda_2 + \Gamma_{13}\lambda_3 \tag{2.80}$$

$$i_2 = \Gamma_{12}\lambda_1 + \Gamma_{22}\lambda_2 + \Gamma_{23}\lambda_3 \tag{2.81}$$

$$i_3 = \Gamma_{13}\lambda_1 + \Gamma_{23}\lambda_2 + \Gamma_{33}\lambda_3 \tag{2.82}$$

The matrix $\mathbf{\Gamma}$ is the inverse of the inductance matrix $\mathbf{L}$:

$$\Gamma = \mathrm{L}^{-1} \tag{2.83}$$

2.12 DOUBLY EXCITED SYSTEMS

In practice, rotating electric machines are characterized by more than one exciting winding. In the system shown in Figure 2.25, a coil on the stator is fed by an electric energy source 1 and a second coil is mounted on the rotor and fed by source 2. For this doubly excited system, we write the relation between flux linkages and currents as

$$\lambda_1 = L_{11}(\theta) i_1 + M(\theta) i_2 \tag{2.84}$$

$$\lambda_2 = M(\theta) i_1 + L_{22}(\theta) i_2 \tag{2.85}$$

The self-inductances L_{11} and L_{22} and the mutual inductance M are given as functions of θ as follows:

$$L_{11}(\theta) = L_1 + \Delta L_1 \cos 2\theta \tag{2.86}$$

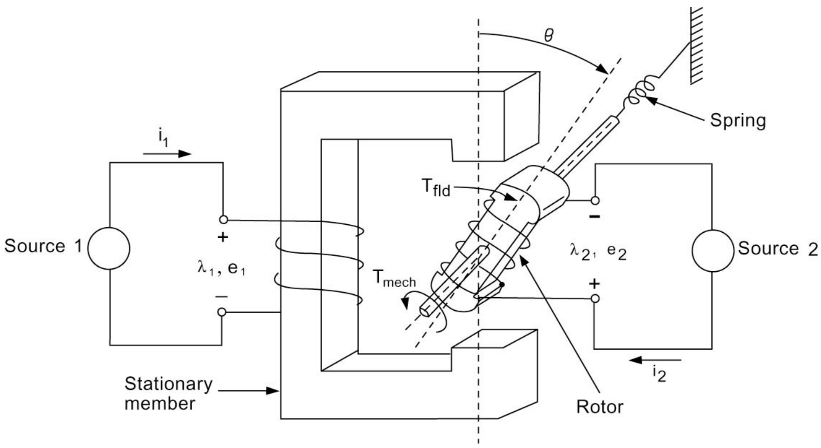


Figure 2.25 Doubly Excited Electromechanical Energy Conversion Device.

where

$$L_1 = \tfrac{1}{2}(L_{\max} + L_{\min}) \tag{2.87}$$

$$\Delta L_1 = \frac{1}{2}(L_{\max} - L_{\min}) \tag{2.88}$$

$$M(\theta) = M_0 \cos\theta \tag{2.89}$$

$$L_{22}(\theta) = L_2 + \Delta L_2 \cos 2\theta \tag{2.90}$$

In many practical applications, ΔL_2 is considerably less than L_2 and we may conclude that L_{22} is independent of the rotor position.

$$T_{\text{fld}} = -[(i_1^2 \Delta L_1 + i_2^2 \Delta L_2)\sin 2\theta + i_1 i_2 M_0 \sin\theta)] \tag{2.91}$$

Let us define

$$T_R = i_1^2 \Delta L_1 + i_2^2 \Delta L_2 \tag{2.92}$$

$$T_M = i_1 i_2 M_0 \tag{2.93}$$

Thus the torque developed by the field is written as

$$T_{\text{fld}} = -(T_M \sin\theta + T_R \sin 2\theta) \tag{2.94}$$

We note that for a round rotor, the reluctance of the air gap is constant and hence the self-inductances L_{11} and L_{22} are constant, with the result that ΔL_1 =

$\Delta L_2 = 0$. We therefore see that for a round rotor $T_R = 0$, and in this case

$$T_{\text{fld}} = -T_M \sin\theta \tag{2.95}$$

For an unsymmetrical rotor the torque is made up of a reluctance torque $T_R \sin 2\theta$ and the primary torque $T_M \sin\theta$.

2.13 SALIENT-POLE MACHINES

The majority of electromechanical energy conversion devices used in present-day applications are in the rotating electric machinery category with symmetrical stator structure. From a broad geometric configuration point of view, such machines can be classified as being either of the salient-pole type, as this class is a simple extension of the discussion of the preceding section, or round-rotor.

In a salient-pole machine, one member (the rotor in our discussion) has protruding or salient poles and thus the air gap between stator and rotor is not uniform, as shown in Figure 2.26. It is clear that results of Section 2.12 are applicable here and we simply modify these results to conform with common machine terminology, shown in Figure 2.26. Subscript 1 is replaced by s to represent stator quantities and subscript 2 is replaced by r to represent rotor quantities. Thus we rewrite Eq. (2.84) as

$$\lambda_s = (L_s + \Delta L_s \cos 2\theta) i_s + (M_0 \cos\theta) i_r \tag{2.96}$$

Similarly, Eq. (2.85) is rewritten as

$$\lambda_r = (M_0 \cos\theta) i_s + L_r i_r \tag{2.97}$$

Note that we assume that L_{22} is independent of θ and is represented by L_r. Thus, $\Delta L_2 = 0$ under this assumption. The developed torque given by Eq. (2.91) is therefore written as

$$T_{\text{fld}} = -i_s i_r M_0 \sin\theta - i_s^2 \Delta L_s \sin 2\theta \tag{2.98}$$

We define the primary or main torque T_1 by

$$T_1 = -i_s i_r M_0 \sin\theta \tag{2.99}$$

We also define the reluctance torque T_2 by

$$T_2 = -i_s^2 \Delta L_s \sin 2\theta \tag{2.100}$$

Thus we have

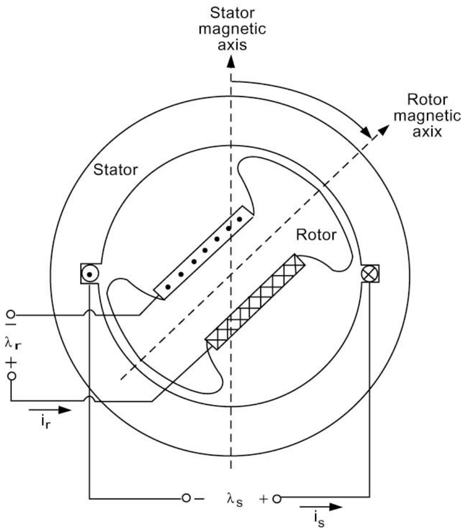


Figure 2.26 Two-Pole Single-Phase Salient-Pole Machine With Saliency On The Rotor.

$$T_{\text{fld}} = T_1 + T_2 \tag{2.101}$$

Let us assume that the source currents are sinusoidal.

$$i_s(t) = I_s \sin \omega_s t \tag{2.102}$$

$$i_r(t) = I_r \sin \omega_r t \tag{2.103}$$

Assume also that the rotor is rotating at an angular speed ω_m and hence,

$$\theta(t) = \omega_m t + \theta_0 \tag{2.104}$$

We examine the nature of the instantaneous torque developed under these conditions.

The primary or main torque T_1, expressed by Eq. (2.99), reduces to the following form under the assumptions of Eqs. (2.102) to (2.104):

$$\begin{aligned} T_1 = -\frac{I_s I_r M_0}{4}\{[(\omega_m + \omega_s - \omega_r)t + \theta_0] + \sin[(\omega_m - \omega_s + \omega_r)t + \theta_0] \\ - \sin[(\omega_m + \omega_s + \omega_r)t + \theta_0] - \sin[(\omega_m - \omega_s - \omega_r)t + \theta_0]]\} \end{aligned} \tag{2.105}$$

An important characteristic of an electric machine is the average torque developed. Examining Eq. (2.105), we note that T_1 is made of four sinusoidal components each of zero average value if the coefficient of t is different from zero. It thus follows that as a condition for nonzero average of T_1, we must satisfy one of the following:

$$\omega_m = \pm\omega_s \pm \omega_r \tag{2.106}$$

For example, when

$$\omega_m = -\omega_s + \omega_r$$

then

$$T_{1_{av}} = -\frac{I_s I_r M_0}{4} \sin\theta_0$$

and when

$$\omega_m = \omega_s + \omega_r$$

then

$$T_{1_{av}} = -\frac{I_s I_r M_0}{4} \sin\theta_0$$

The reluctance torque T_2 of Eq. (2.100) can be written using Eqs. (2.102) to (2.104) as

$$\begin{aligned} T_2 = -\frac{I_s^2 \Delta L_s}{4}\{&2\sin(2\omega_m t + 2\theta_0) - \sin[2(\omega_m + \omega_s)t + 2\theta_0] \\ &- \sin[2(\omega_m - \omega_s)t + 2\theta_0]\} \end{aligned} \tag{2.107}$$

The reluctance torque will have an average value for

$$\omega_m = \pm\omega_s \tag{2.108}$$

When either of the two conditions is satisfied,

$$T_{2_{av}} = \frac{I_s^2 \Delta L_2}{4} \sin 2\theta_0$$

2.14 ROUND OR SMOOTH AIR-GAP MACHINES

A round-rotor machine is a special case of salient-pole machine where the air gap between the stator and rotor is (relatively) uniform. The term *smooth air gap* is an idealization of the situation illustrated in Figure 2.27. It is clear that for the case of a smooth air-gap machine the term ΔL_s is zero, as the reluctance does not vary with the angular displacement θ. Therefore, for the machine of Figure 2.27, we have

$$\lambda_s = L_s i_s + M_0 \cos\theta i_r \tag{2.109}$$

$$\lambda_r = M_0 \cos\theta i_s + L_r i_r \tag{2.110}$$

Under the assumptions of Eqs. (2.102) to (2.104), we obtain

$$T_{\text{fld}} = T_1 \tag{2.111}$$

where T_1 is as defined in Eq. (2.105).

We have concluded that for an average value of T_1 to exist, one of the conditions of Eq. (2.106) must be satisfied:

$$\omega_m = \pm\omega_s \pm \omega_r \tag{2.112}$$

We have seen that for

$$\omega_m = -\omega_s + \omega_r \tag{2.113}$$

then

$$T_{\text{av}} = -\frac{I_s I_r M_0}{4} \sin\theta \tag{2.114}$$

Now substituting Eq. (2.113) in to Eq. (2.105), we get

$$\begin{aligned} T_{fld} = -\frac{I_s I_r M_0}{4} \{ &\sin\theta_0 + \sin[2(\omega_r - \omega_s)t + \theta_0] - \sin(2\omega_r t + \theta_0) \\ &- \sin(-2\omega_s t + \theta_0) \} \end{aligned} \tag{2.115}$$

The first term is a constant, whereas the other three terms are still sinusoidal time functions and each represents an alternating torque. Although these terms are of zero average value, they can cause speed pulsations and vibrations that may be harmful to the machine's operation and life. The alternating torques can be eliminated by adding additional windings to the stator and rotor, as discussed presently.

Two-Phase Machines

Consider the machine of Figure 2.28, where each of the distributed windings is represented by a single coil. It is clear that this is an extension of the machine of Figure 2.27 by adding one additional stator winding (*bs*) and one

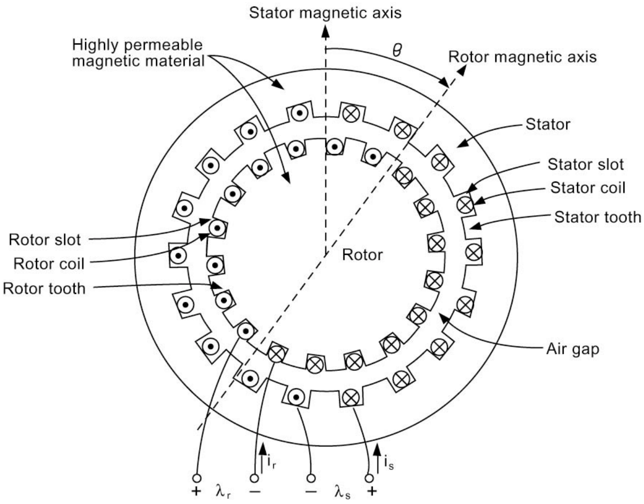


Figure 2.27 Smooth Air-Gap Machine.

additional rotor winding (*br*) with the relative orientation shown in Figure 2.28(B). Our analysis of this machine requires first setting up the inductances required. This can be best done using vector terminology. We can write for this four-winding system:

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{ar} \\ \lambda_{bs} \\ \lambda_{br} \end{bmatrix} = \left[\begin{array}{cc|cc} L_s & M_0\cos\theta & 0 & M_0\sin\theta \\ M_0\cos\theta & L_r & -M_0\sin\theta & 0 \\ \hline 0 & M_0\sin\theta & L_s & M_0\cos\theta \\ -M_0\sin\theta & 0 & M_0\cos\theta & L_r \end{array}\right] \begin{bmatrix} i_{as} \\ i_{ar} \\ i_{bs} \\ i_{br} \end{bmatrix} \quad \textbf{(2.116)}$$

The field energy is the same as given by Eq. (2.108). The torque is obtained in the usual manner. Let us now assume that the terminal currents are given by the balanced, two-phase current sources

$$i_{as} = I_s \cos\omega_s t \quad \textbf{(2.117)}$$

$$i_{bs} = I_s \sin\omega_s t \quad \textbf{(2.118)}$$

$$i_{ar} = I_r \cos\omega_r t \quad \textbf{(2.119)}$$

$$i_{br} = I_r sn\omega_r t \quad \textbf{(2.120)}$$

We also assume that

$$\theta(t) = \omega_m t + \theta_0 \quad \textbf{(2.121)}$$

The torque is given by

$$T_{\text{fld}} = M_0[(i_{ar}i_{bs} - i_{br}i_{as})\cos\theta - (i_{ar}i_{as} + i_{br}i_{bs})\sin\theta] \quad \textbf{(2.122)}$$

Substituting Eqs. (2.117) through (2.121) into (2.122), we obtain (after some manipulations)

$$T_{\text{fld}} = M_0 I_s I_r \sin[(\omega_m - \omega_s - \omega_r)t + \theta_0] \quad \textbf{(2.123)}$$

The condition for nonzero average torque is given by

$$\omega_m = \omega_s - \omega_r \quad \textbf{(2.124)}$$

For this condition, we have

$$T_{\text{fld}} = M_0 I_s I_r \sin\theta_0 \quad \textbf{(2.125)}$$

The instantaneous torque in this case is constant in spite of the excitation being sinusoidal.

2.15 MACHINE-TYPE CLASSIFICATION

The results of the preceding provides a basis for defining conventional machine types.

Synchronous Machines

The two-phase machine of Figure 2.29 is excited with direct current applied to the rotor ($\omega_r = 0$) and balanced two-phase currents of frequency ω_s applied to the stator. With $\omega_r = 0$ we get

$$\omega_m = \omega_s \quad \textbf{(2.126)}$$

Thus, the rotor of the machine should be running at the single value defined by the stator sources to produce a torque with nonzero average value. This mode of operation yields a synchronous machine which is so named because is can convert average power only at one mechanical speed – the synchronous speed, ω_s. The synchronous machine is the main source of electric energy in modern power systems acting as a generator.

Induction Machines

Single-frequency alternating currents are fed into the stator circuits and the rotor circuits are all short circuited in a conventional induction machine. The machine in Figure 2.29 is used again for the analysis. Equations (2.117)

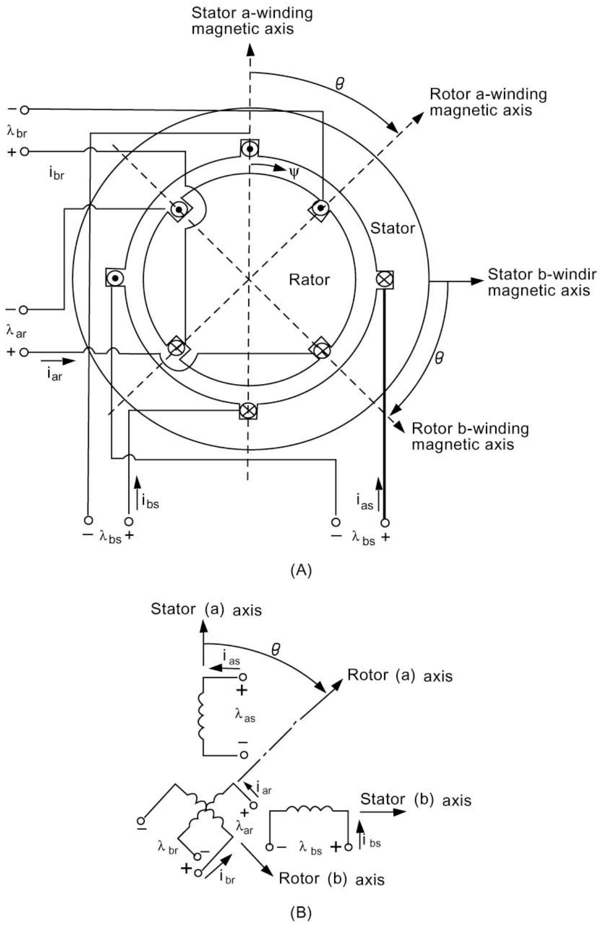


Figure 2.28 Two-Phase Smooth Air-Gap Machine.

and (2.118) still apply and are repeated here:

$$i_{a_s} = I_s \cos \omega_s t \tag{2.127}$$

$$i_{b_s} = I_s \sin \omega_s t \tag{2.128}$$

With the rotor circuits short-circuited,

$$\upsilon_{ar} = \upsilon_{br} = 0 \tag{2.129}$$

and the rotor is running according to Eq. (2.121):

$$\theta(t) = \omega_m t + \theta_0 \tag{2.130}$$

Conditions (2.129) are written as

$$\upsilon_{ar} = R_r i_{ar} + \frac{d\lambda_{ar}}{dt} = 0 \tag{2.131}$$

$$\upsilon_{br} = R_r i_{br} + \frac{d\lambda_{br}}{dt} = 0 \tag{2.132}$$

Here we assume that each rotor phase has a resistance of $R_r \Omega$. We have, by Eq. (2.116),

$$\lambda_{ar} = M_0 \cos\theta i_{as} + L_r i_{ar} + M_0 \sin\theta i_{bs} \tag{2.133}$$

$$\lambda_{br} = -M_0 \sin\theta i_{as} + M_0 \cos\theta i_{bs} + L_r i_{br} \tag{2.134}$$

As a result, we have

$$0 = R_r i_{ar} + L_r \frac{di_{ar}}{dt} + M_0 I_s \frac{d}{dt}[\cos\omega_s t \cos(\omega_m t + \theta_0) + \sin\omega_s t \sin(\omega_m t + \theta_0)] \tag{2.135}$$

and

$$0 = R_r i_{br} + L_r \frac{di_{br}}{dt} + M_0 I_s \frac{d}{dt}[-\cos\omega_s t \sin(\omega_m t + \theta_0) + \sin\omega_s t \cos(\omega_m t + \theta_0)] \tag{2.136}$$

A few manipulations provide us with

$$M_0 I_s(\omega_s - \omega_m)\sin[(\omega_s - \omega_m)t - \theta_0] = L_r \frac{di_{ar}}{dt} + R_r i_{ar} \tag{2.137}$$

$$-M_0 I_s(\omega_s - \omega_m)\cos[(\omega_s - \omega_m)t - \theta_0] = L_r \frac{di_{br}}{dt} + R_r i_{br} \tag{2.138}$$

The right-hand sides are identical linear first-order differential operators. The left sides are sinusoidal voltages of equal magnitude by 90° apart in phase. The rotor currents will have a frequency of (ω_s – ω_m), which satisfies condition (2.124), and thus an average power and an average torque will be produced by the induction machine. We emphasize the fact that currents induced in the rotor have a frequency of (ω_s – ω_m) and that average torque can be produced.

2.16 P-POLE MACHINES

The configuration of the magnetic field resulting from coil placement in the magnetic structure determines the number of poles in an electric machine.

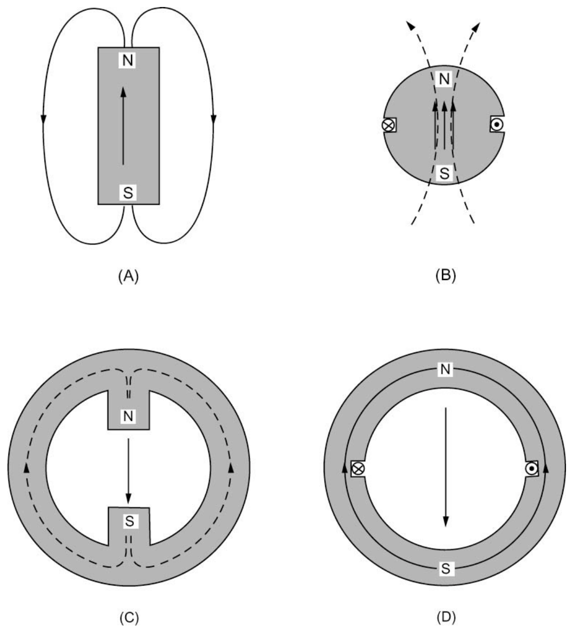


Figure 2.29 Two-Pole Configurations.

An important point to consider is the convention adopted for assigning polarities in schematic diagrams, which is discussed presently. Consider the bar magnet of Figure 2.30(A). The magnetic flux lines are shown as closed loops oriented from the south pole to the north pole within the magnetic material. Figure 2.30(B) shows a two-pole rotor with a single coil with current flowing in the direction indicated by the dot and cross convention. According to the right-hand rule, the flux lines are directed upward inside the rotor material, and as a result we assert that the south pole of the electromagnet is on the bottom part and that the north pole is at the top, as shown.

The situation with a two-pole stator is explained in Figure 2.30(C) and 2.30(D). First consider 2.30(C), showing a permanent magnet shaped as shown. According to our convention, the flux lines are oriented away from the south pole toward the north pole within the magnetic material (not in air gaps). For 2.30(D), we have an electromagnet resulting from the insertion of a single coil in slots on the periphery of the stator as shown. The flux lines are oriented in accordance with the right-hand rule and we conclude that the north and south pole orientations are as shown in the figure.

Consider now the situation illustrated in Figure 2.31, where two coils are connected in series and placed on the periphery of the stator in part (a) and on the rotor in part (b). An extension of the prior arguments concerning a two-pole machine results from the combination of the stator and rotor of Figure 2.31 and is shown in Figure 2.32 to illustrate the orientation of the magnetic axes of rotor and stator.

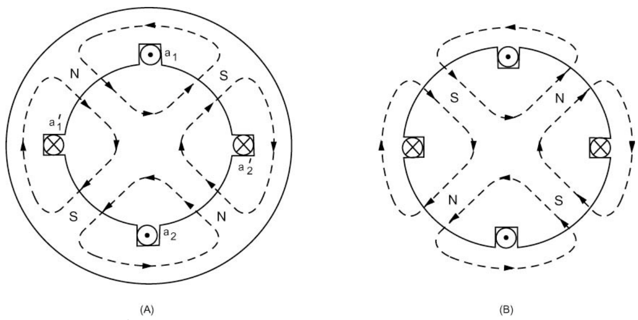


Figure 2.30 Four-pole configurations: (A) stator arrangement, and (B) rotor arrangement.

It is clear that any arbitrary even number of poles can be achieved by placing the coils of a given phase in symmetry around the periphery of stator and rotor of a given machine. The number of poles is simply the number encountered in one round trip around the periphery of the air gap. It is necessary for successful operation of the machine to have the same number of poles on the stator and rotor.

Consider the four-pole, single-phase machine of Figure 2.32. Because of the symmetries involved, the mutual inductance can be seen to be

$$M(\theta) = M_0 \cos 2\theta \tag{2.139}$$

Compared with Eq. (2.89) for a two-pole machine, we can immediately assert that for a *P*-pole machine,

$$M(\theta) = M_0 \cos \frac{P\theta}{2} \tag{2.140}$$

where *P* is the number of poles.

We note here that our treatment of the electric machines was focused on two-pole configurations. It is clear that extending our analytic results to a *P*-pole machine can easily be done by replacing the mechanical angle θ in a relation developed for a two-pole machine by the angle $P\theta/2$ to arrive at the corresponding relation for a *P*-pole machine. As an example, Eqs. (2.109) and (2.110) for a *P*-pole machine are written as

$$\lambda_s = L_s i_s + \left(M_0 \cos \frac{P\theta}{2} \right) i_r \qquad \textbf{(2.140)}$$

$$\lambda_r = \left(M_0 \cos \frac{P\theta}{2} \right) i_s + L_r i_r \tag{2.141}$$

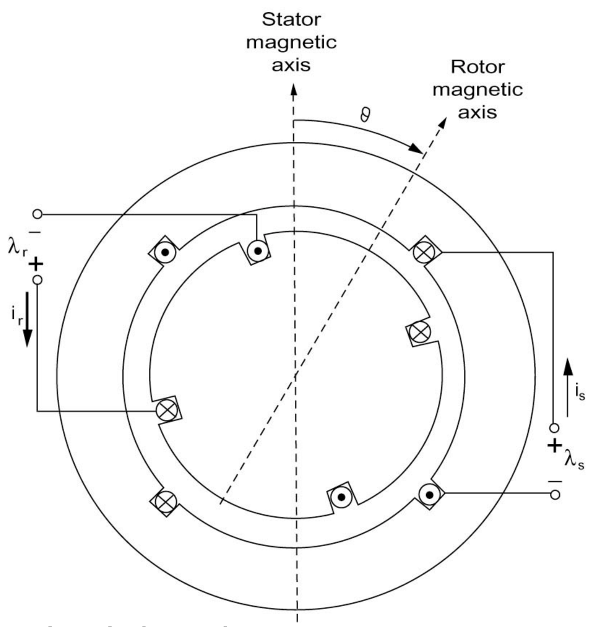


Figure 2.31 Four-Pole Single-Phase Machine.

Similarly, the torque expression in Eq. (2.99) becomes

$$T_1 = -i_s i_r M_0 \sin\frac{P\theta}{2} \tag{2.142}$$

Note that θ in the expressions above is in mechanical degrees.

The torque T_1 under the sinusoidal excitation conditions (2.109) and (2.110) given by Eq. (2.112) is rewritten for a P-pole machine as

$$\begin{aligned} T_1 = -\frac{I_s I_r M_0}{4}\Bigg\{ &\sin\left[\left(\frac{P\omega_m}{2} + \omega_s - \omega_r\right)t + \frac{P\theta_0}{2}\right] \\ &+ \sin\left[\left(\frac{P\omega_m}{2} - \omega_2 + \omega_r\right)t + \frac{P\theta_0}{2}\right] \\ &- \sin\left[\left(\frac{P\omega_m}{2} + \omega_s + \omega_r\right)t + \frac{P\theta_0}{2}\right] \\ &- \sin\left[\left(\frac{P\omega_m}{2} - \omega_s - \omega_r\right)t + \frac{P\theta_0}{2}\right]\Bigg\} \end{aligned} \tag{2.144}$$

The conditions for average torque production of Eq. (2.113) are written for a P-pole machine as

$$\omega_m = \frac{2}{P}(\pm\omega_s \pm \omega_r) \qquad \textbf{(2.145)}$$

Thus, for given electrical frequencies the mechanical speed is reduced as the number of poles is increased.

A time saving and intuitively appealing concept in dealing with *P*-pole machines is that of electrical degrees. Let us define the angle θ_e corresponding to take a mechanical angle θ, in a *P*-pole machine the

$$\theta_e = \frac{P}{2}\theta \qquad \textbf{(2.146)}$$

With this definition we see that all statements, including θ for a two-pole machine apply to any *P*-pole machine with θ taken as an electrical angle.

Consider the first condition of Eq. (2.145) with $\omega_r = 0$ corresponding to synchronous machine operation:

$$\omega_m = \frac{2}{P}\omega_s \qquad \textbf{(2.147)}$$

The stator angular speed ω_s is related to frequency f_s in hertz by

$$\omega_s = 2\pi f_s \qquad \textbf{(2.148)}$$

The mechanical angular speed ω_m is related to the mechanical speed n in revolutions per minute by

$$\omega_m = \frac{2\pi n}{60} \qquad \textbf{(2.149)}$$

Combining Eq. (2.147) with Eq. (2.149), we obtain

$$f_s = \frac{Pn}{120} \qquad \textbf{(2.150)}$$

This is an important relation in the analysis of rotating electrical machines.

2.17 POWER SYSTEM REPRESENTATION

A major portion of the modern power system utilizes three-phase ac circuits and devices. It is clear that a detailed representation of each of the three phases in the system is cumbersome and can also obscure information about the system. A balanced three-phase system is solved as a single-phase circuit made of one line and the neutral return; thus a simpler representation would involve retaining one line to represent the three phases and omitting the neutral. Standard symbols are used to indicate the various components. A transmission line is represented by a single line between two ends. The simplified diagram is called the *single-line diagram*.

The one-line diagram summarizes the relevant information about the system for the particular problem studied. For example, relays and circuit breakers are not important when dealing with a normal state problem. However, when fault conditions are considered, the location of relays and circuit breakers is important and is thus included in the single-line diagram.

The International Electrotechnical Commission (IEC), the American National Standards Institute (ANSI), and the Institute of Electrical and Electronics Engineers (IEEE) have published a set of standard symbols for electrical diagrams. A basic symbol for a rotating machine is a circle. Figure 2.32(A) shows rotating machine symbols. If the winding connection is desired, the connection symbols may be shown in the basic circle using the representations given in Figure 2.32(B). The symbols commonly used for transformer representation are given in Figure 2.33(A). The two-circle symbol is the symbol to be used on schematics for equipment having international usage according to IEC. Figure 2.33(B) shows symbols for a number of single-phase transformers, and Figure 2.34 shows both single-line symbols and three-line symbols for three-phase transformers.

PROBLEMS

Problem 2.1

In the circuit shown in Figure 2.35, the source phasor voltage is $V = 30\angle 15^\circ$. Determine the phasor currents I_2 and I_3 and the impedance Z_2. Assume that I_1 is equal to five *A*. Calculate the apparent power produced by the source and the individual apparent powers consumed by the 1-ohm resistor, the impedance Z_2, and the resistance R_3. Show that conservation of power holds true.

Problem 2.2

A three-phase transmission link is rated 100 kVA at 2300 V. When operating at rated load, the total resistive and reactive voltage drops in the link are, respectively, 2.4 and 3.6 % of the rated voltage. Determine the input power and power factor when the link delivers 60 kW at 0.8 PF lagging at 2300 V.

Problem 2.3

A 60-hp, three-phase, 440-V induction motor operates at 0.8 PF lagging.

a) Find the active, reactive, and apparent power consumed per phase.
b) Suppose the motor is supplied from a 440-V source through a feeder whose impedance is $0.5 + j0.3$ ohm per phase. Calculate the

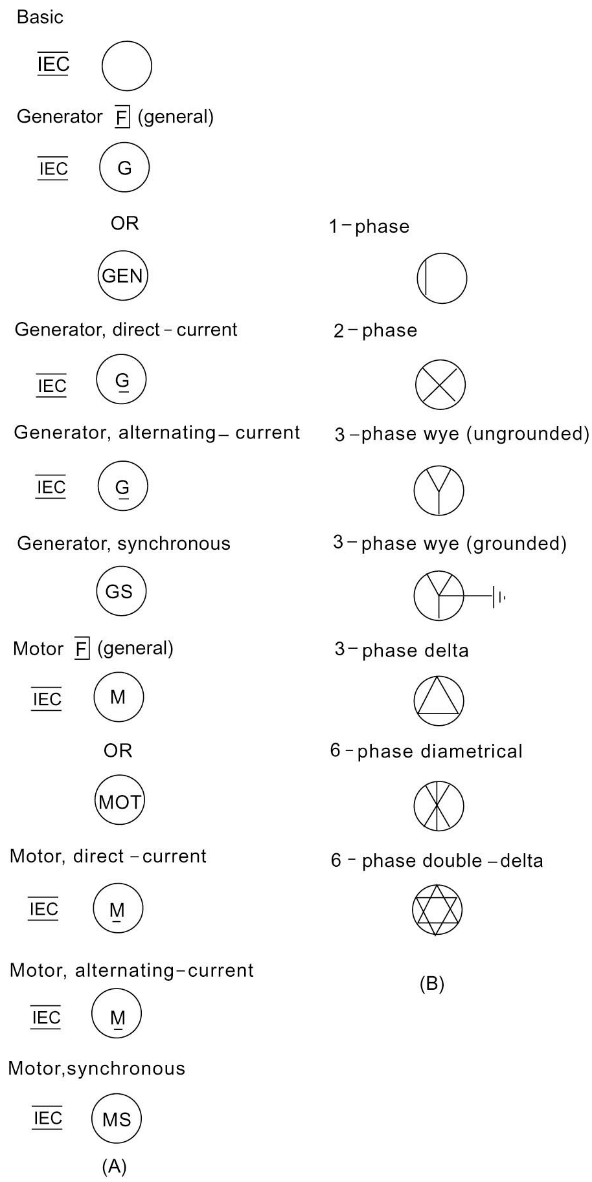


Figure 2.32 Symbols for Rotating Machines (A) and Their Winding Connections (B).

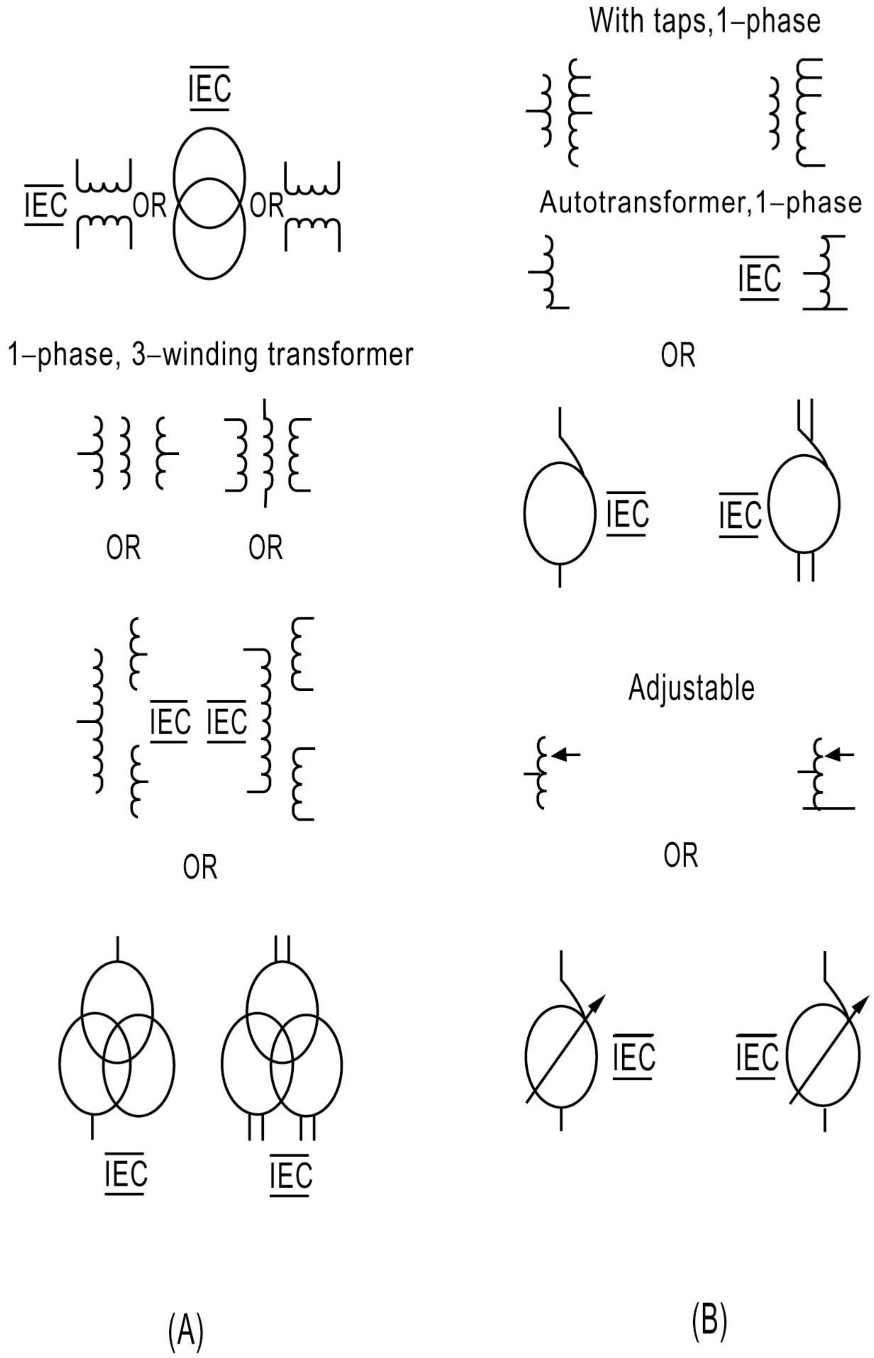


Figure 2.33 (A) Transformer Symbols, and (B) Symbols for Single-Phase Transformers.

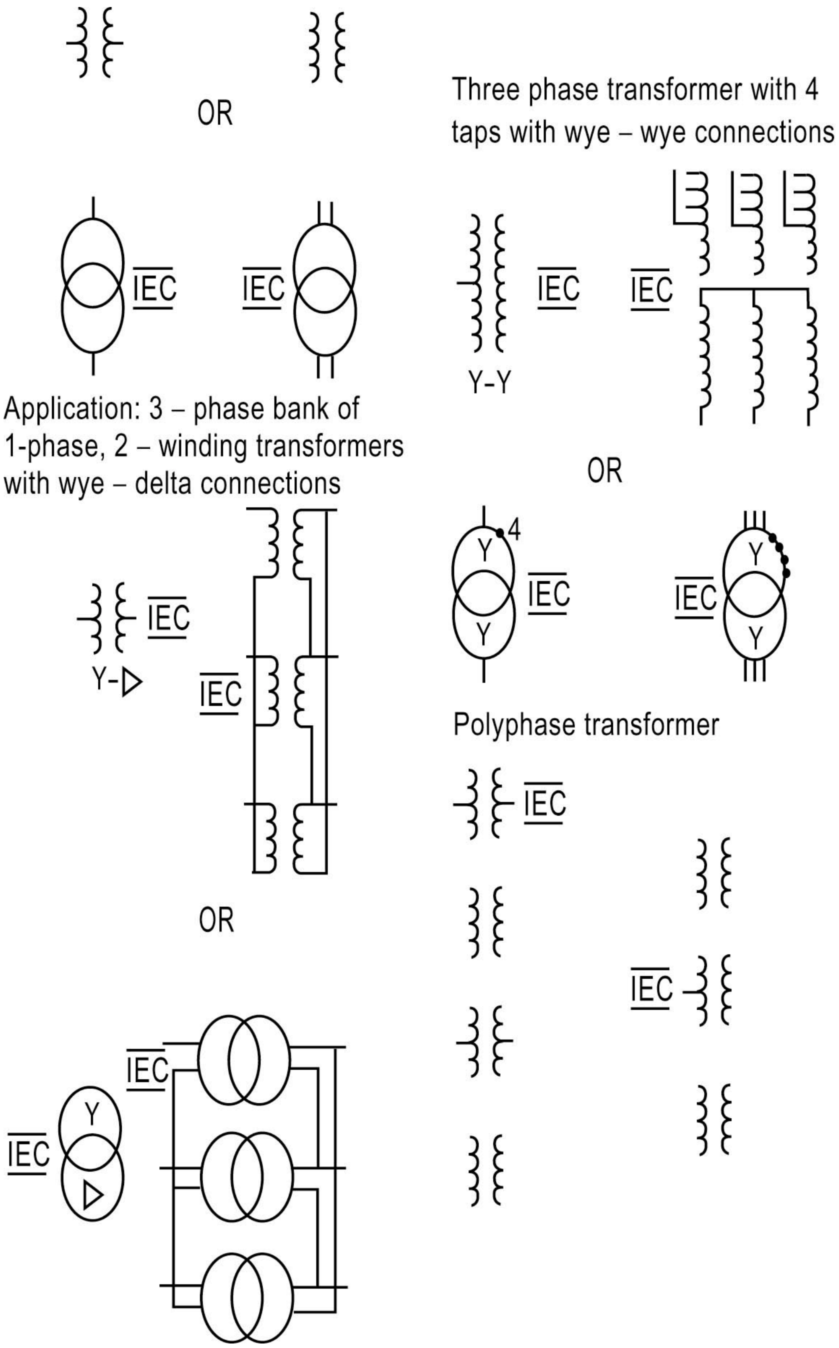


Figure 2.34 Symbols for Three-Phase Transformers.

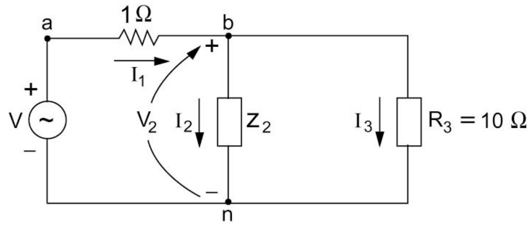


Figure 2.35 Circuit for Problem 2.1.

c) voltage at the motor side, the source power factor, and the efficiency of transmission.

Problem 2.4
Repeat Problem 2.3 if the motor's efficiency is 85%.

Problem 2.5
Repeat Problem 2.4 if the PF is 0.7 lagging.

Problem 2.6
Consider a 100 kW load operating at a lagging power factor of 0.7. A capacitor is connected in parallel with the load to raise the source power factor to 0.9 p.f. lagging. Find the reactive power supplied by the capacitor assuming that the voltage remains constant.

Problem 2.7
A balanced Y-connected 3 phase source with voltage $V_{ab} = 240\angle 0^\circ$ V is connected to a balanced Δ load with $Z_\Delta = 30\angle 35^\circ\ \Omega$. Find the currents in each of the load phases and hence obtain the current through each phase of the source.

Problem 2.8
Assume that the load of Problem 2.7 is connected to the source using a line whose impedance is $Z_L = 1\angle 80^\circ\ \Omega$ for each phase. Calculate the line currents, the Δ-load currents, and the voltages at the load terminals.

Problem 2.9
A balanced, three-phase 240-V source supplies a balanced three-phase load. If the line current I_A is measured to be 5 A, and is in phase with the line-to-line voltage V_{BC}, find the per phase load impedance if the load is (a) Y-connected, and (b) Δ-connected.

Problem 2.10
Two balanced Y-connected loads, one drawing 20 kW at 0.8 p.f. lagging and the other 30 kW at 0.9 p.f. leading, are connected in parallel and supplied by a balanced three-phase Y-connected, 480-V source. Determine the impedance per phase of each load and the source currents.

Problem 2.11

A load of 30 MW at 0.8 p.f. lagging is served by two lines from two generating sources. Source 1 supplies 15 MW at 0.8 p.f. lagging with a terminal voltage of 4600 V line-to-line. The line impedance is $(1.4 + j1.6)\ \Omega$ per phase between source 1 and the load, and $(0.8 + j1)\ \Omega$ per phase between source 2 and the load. Find

a) The voltage at the load terminals
b) The voltage at the terminals of source 2, and
c) The active and reactive power supplied by source 2.

Problem 2.12

The impedance of a three-phase line is $0.3 + j2.4$ per phase. The line feeds two balanced three-phase loads connected in parallel. The first load takes 600 kVA at 0.7 p.f. lagging. The second takes 150 kW at unity power factor. The line to line voltage at the load end of the line is 3810.5 V. Find

a) The magnitude of the line voltage at the source end of the line.
b) The total active and reactive power loss in the line.
c) The active and reactive power supplied at the sending end of the line.

Problem 2.13

Three loads are connected in parallel across a 12.47 kV three-phase supply. The first is a resistive 60 kW load, the second is a motor (inductive) load of 60 kW and 660 kvar, and the third is a capacitive load drawing 240 kW at 0.8 p.f. Find the total apparent power, power factor, and supply current.

Problem 2.14

A Y-connected capacitor bank is connected in parallel with the loads of Problem 2.13. Find

a) The total kvar and capacitance per phase in μF to improve the overall power factor to 0.8 lagging.
b) The corresponding line current.

Problem 2.15

Assume that 30 V and 5 A are chosen as base voltage and current for the circuit of Problem 2.1.

a) Find the corresponding base impedance and VA.
b) Find the phasor currents I_2 and I_3 in per unit.
c) Determine the source apparent power in per unit.

Problem 2.16

Consider the transmission link of Problem 2.2 and choose 100 kVA and 2300 V as base kVA and voltage. Determine the input power in per unit under the conditions of Problem 2.2.

Problem 2.17
Assume for the motor of Problem 2.3 that 50 kW and 440 V are taken as base values. Find the voltage in per unit at the motor side.

Problem 2.18
Assume that the base voltage is 4600 V in the system of Problem 2.11, and that 50 MVA is the corresponding apparent power base. Repeat Problem 2.11 using per unit values.

Problem 2.19
Repeat Problem 2.12 using per unit values assuming that 1000 kVA is base apparent power and 3 Ω is the base impedance.

Problem 2.20
The following information is available about a 40 MVA 20-kV/400 kV, single-phase transformer:

$$Z_1 = 0.9 + j1.8\ \Omega$$
$$Z_2 = 128 + j288\ \Omega$$

Using the transformer rating as base, determine the per unit impedance of the transformer from the ohmic value referred to the low voltage side. Find the per unit impedance using the ohmic value referred to the high voltage side.

Problem 2.21
Consider a toroidal coil with relative permeability of 1500 with a circular cross section whose radius is 0.025 cm. The outside radius of the toroid is 0.2 cm. Find the inductance of the coil assuming that $N = 10$ turns.

Problem 2.22
The eddy-current and hysteresis losses in a transformer are 450 and 550 W, respectively, when operating from a 60-Hz supply with an increase of 10% in flux density. Find the change in core losses.

Problem 2.23
The relationship between current, displacement, and flux linkages in a conservative electromechanical device is given by

$$i = \lambda[0.7\lambda + 2.0(x-1)^2]$$

Find expressions for the stored energy and the magnetic field force in terms of λ and x. Find the force for $x = 0.9$.

Problem 2.24
Repeat Problem 2.23 for the relationship

$$\mathrm{i} = \lambda^3 + \lambda(0.2\lambda + 0.9x)$$

Problem 2.25
A plunger-type solenoid is characterized by the relation

$$\lambda = \frac{8i}{1+10^4 x/2.54}$$

Find the force exerted by the field for $x = 2.54 \times 10^{-3}$ and $i = 12$ A.

Problem 2.26
The inductance of a coil used with a plunger-type electromechanical device is given by

$$L = \frac{1.75\times 10^{-5}}{x}$$

where x is the plunger displacement. Assume that the current in the coil is given by

$$i(t) = 8\sin\omega t$$

where $\omega = 2\pi\,(60)$. Find the force exerted by the field for $x = 10^{-2}$ m. Assume that x is fixed and find the necessary voltage applied to the coil terminals given that its resistance is 1 Ω.

Problem 2.27
A rotating electromechanical conversion device has a stator and rotor, each with a single coil. The inductances of the device are

$L_{11} = 0.5$ H $\qquad$ $L_{22} = 2.5$ H
$L_{12} = 1.25 \cos\theta$ H

Where the subscript 1 refers to stator and the subscript 2 refers to rotor. The angle θ is the rotor angular displacement from the stator coil axis. Express the torque as a function of currents i_1, i_2 and θ and compute the torque for $i_1 = 3$ A and $i_2 = 1$ A.

Problem 2.28
Assume for the device of Problem 2.27 that

$$L_{11} = 0.3 + 0.2\cos 2\theta$$

All other parameters are unchanged. Find the torque in terms of θ for $i_1 = 2.5$ A and (a) $i_2 = 0$; (b) $i_2 = 1.5$ A.

Problem 2.29
Assume for the device of Problem 2.27 that the stator and rotor coils are connected in series, with the current being

$$i(t) = I_m \sin \omega t$$

Find the instantaneous torque and its average value over one cycle of the supply current in terms of I_m and ω.

Problem 2.30

A rotating electromechanical energy conversion device has the following inductances in terms of θ in radians (angle between rotor and stator axes):

$$L_{11} = 0.8\theta$$
$$L_{22} = -0.25 + 1.8\theta$$
$$L_{12} = -0.75 + 1.4\theta$$

Find the torque developed for the following excitations.

a) $i_1 = 15$ A, $i_2 = 0$.
b) $i_1 = 0$ A, $i_2 = 15$ A.
c) $i_1 = 15$ A, $i_2 = 15$ A.
d) $i_1 = 15$ A, $i_2 = -15$ A.

Problem 2.31

For the machine of Problem 2.27, assume that the rotor coil terminals are shorted ($e_2 = 0$) and that the stator current is given by

$$i_1(t) = I \sin \omega t$$

Find the torque developed as a function of I, θ, and time.

Problem 2.32

For the device of Problem 2.31, the rotor coil terminals are connected to a 10-Ω resistor. Find the rotor current in the steady state and the torque developed.