

# Fundamentals of Electromagnetics

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## 1.1 RF AND MICROWAVE FREQUENCY RANGES

The rapid technological advances in electronics, electro-optics, and computer science have profoundly affected our everyday lives. They have also set the stage for an unprecedented drive toward the improvement of existing medical devices and the development of new ones. In particular, the advances in radio-frequency (RF)/microwave technology and computation techniques, among others, have paved the way for exciting new therapeutic and diagnostic methods. Frequencies, from RF as low as 400 kHz through microwave frequencies as high as 10 GHz, are presently being investigated for therapeutic applications in areas such as cardiology, urology, surgery, ophthalmology, cancer therapy, and others and for diagnostic applications in cancer detection, organ imaging, and more.

At the same time, safety concerns regarding the biological effects of electromagnetic (EM) radiation have been raised, in particular at a low level of exposure. A variety of waves and signals have to be considered, from pure or almost pure sine waves to digital signals, such as in digital radio, digital television, and digital mobile phone systems. The field has become rather sophisticated, and establishing safety recommendations or rules and making adequate measurements require quite an expertise.

In this book, we limit ourselves to the effects and applications of RF and microwave fields. This covers a frequency range from about 100 kHz to 10 GHz and above. This choice is appropriate, although effects at RF and microwaves,

respectively, are of a different nature. It excludes low-frequency (LF) and extremely low frequency (ELF) effects, which do not involve any radiation. It also excludes ultraviolet (UV) and X-rays, called *ionizing* because they can disrupt molecular or atom structures. The RF/microwave frequency range covered here may be called *nonionizing*.

Radiation is a phenomenon characterizing the RF/microwave range. It is well known that structures radiate poorly when they are small with respect to the wavelength. For example, the wavelengths at the power distribution frequencies of 50 and 60 Hz are 6.000 and 5.000 km, respectively, which are enormous with respect to the objects we use in our day-to-day life. In fact, to radiate efficiently, a structure has to be large enough with respect to the wavelength  $\lambda$ . The concepts of radiation, antennas, far field, and near field have to be investigated.

On the other hand, at RF and microwave frequencies, the electric ( $E$ ) and magnetic ( $H$ ) fields are simultaneously present: if there is an electric field, then there is a coupled magnetic field and vice versa. If one is known, the other can be calculated: They are linked together by the well-known Maxwell's equations. Later in this book, we shall be able to separate some biological effects due to one field from some due to the other field. We need, however, to remember that we are considering the general case, which is that of the complete field, called the EM field. Hence, we are not considering direct-current (DC) and LF electric or magnetic fields into tissue.

Because we limit ourselves to the RF/microwave range, we may refer to our subject of interaction of electric and magnetic fields with organic matter as *biological effects of nonionizing radiation*. It should be well noticed that, by specifically considering a frequency range, we decide to describe the phenomena in what is called the *frequency domain*, that is, when the materials and systems of interest are submitted to a source of sinusoidal fields. To investigate properties over a frequency range, wide or narrow, we need to change the frequency of the source. The frequency domain is not "physical" because a sinusoidal source is not physical: It started to exist an infinite amount of time ago and it lasts forever. Furthermore, the general description in the frequency domain implies complex quantities, with a real and an imaginary part, respectively, which are not physical either. The frequency-domain description is, however, extremely useful because many sources are (almost) monochromatic.

To investigate the actual effect of physical sources, however, one has to operate in what is called the *time domain*, where the phenomena are described as a function of time and hence they are real and physically measurable. Operating in the time domain may be rather difficult with respect to the frequency domain. The interaction of RF/microwave fields with biological tissues is investigated mostly in the frequency domain, with sources considered as sinusoidal. Today numerical signals, such as for telephony, television, and frequency-modulated (FM) radio, may, however, necessitate time-domain analyses and measurements.

There is an interesting feature to note about microwaves: They cover, indeed, the frequency range where the wavelength is of the order of the size of objects of common use, that is, meter, decimeter, centimeter, and millimeter, depending of course on the material in which it is measured. One may, hence, wonder whether such wavelengths can excite resonance in biological tissues and systems. We shall come back later to this question.

## 1.2 FIELDS

Investigating the interaction of EM fields with biological tissues requires a good physical insight and mathematical understanding of what are *fields*. A field is associated with a physical phenomenon present in a given region of space. As an example, the temperature in a room is a field of temperature, composed of the values of temperature in a number of points of the room. One may say the same about the temperature *distribution* inside a human body, for instance. We do not see the field, but it exists, and we can for instance visualize constant-temperature or isothermal surfaces.

There are fields of different nature. First, fields may be either *static* or *time dependent*. Considering, for instance, the temperature field just described, the room may indeed be heated or cooled, which makes the temperature field time dependent. The human body may also be submitted to a variety of external sources or internal reasons which affect the temperature distribution inside the body. In this case, the isothermal surfaces will change their shapes as a function of time.

Second, the nature of the field may be such that one parameter only, such as magnitude, is associated with it. Then, the field is defined as *scalar*. The temperature field, for instance, inside a room or a human body, is a scalar field. One realizes that plotting a field may require skill, and also memory space, if the structure is described in detail or if the observer requires a detailed description of the field in space. This is true even in the simplest cases, when the field is scalar and static.

On the other hand, in a *vector* field, a vector represents both the magnitude and the direction of the physical quantity of interest at points in space, and this vector field may also be static or time dependent. When plotting a static scalar field, that is, one quantity, in points of space already requires some visualization effort. On the other hand, plotting a time-dependent vector field, that is, three time-varying quantities, in points of space obviously requires much more attention. A vector field is described by a set of *direction lines*, also known as *stream lines* or *flux lines*. The direction line is a curve constructed so that the field is tangential to the curve in all points of the curve.

### 1.3 ELECTROMAGNETICS

#### 1.3.1 Electric Field and Flux Density

The electric field  $E$  is derived from Coulomb's law, which expresses the interaction between two electric point charges. Experimentally, it has been shown that

1. Two charges of opposite polarity attract each other, while they repel when they have the same polarity, and hence a charge creates a field of force.
2. The force is proportional to the product of charges.
3. The force acts along the line joining the charges and hence the force field is vectorial.
4. The force is higher when the charges are closer.
5. The force depends upon the electric properties of the medium in which the charges are placed.

The first observations showed that the force is about proportional to the square of the distance between them. In 1936, the difference between the measured value and the value 2 for the exponent was of the order of  $2 \times 10^{-9}$  [1]. It is admitted as a postulate that the exponent of the distance in the law expressing the force between the two charges is exactly equal to 2. It has been demonstrated that this postulate is necessary for deriving Maxwell's equations from a relativistic transformation of Coulomb's law under the assumption that the speed of light is a constant with respect to the observer [2, 3]. Hence, Coulomb's law is

$$\vec{f} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \vec{a}_r \quad \text{N} \quad (1.1)$$

where  $\vec{f}$  is the force;  $q_1$  and  $q_2$  the value of the charges, expressed in *coulombs* (C), including their polarity; the factor  $4\pi$  is due to the use of the rationalized meter-kilogram-second (MKS) system, exhibiting a factor  $4\pi$  when the symmetry is spherical; and  $\epsilon_0$  measures the influence of the medium containing the charges, equal to approximately  $10^{-9}/36\pi$  *farads per meter* ( $\text{F m}^{-1}$ ) in vacuum.

If a test charge  $\Delta q$  is placed in the field of force created by a charge  $q$ , it undergoes a force

$$\vec{f} = \frac{q_1(\Delta q)}{4\pi\epsilon_0 r^2} \vec{a}_r \quad \text{N} \quad (1.2)$$

The test charge  $\Delta q$  is small enough to avoid any perturbation of the field of force created by  $q$ . The intensity of the *electric field*, in *volts per meter* ( $\text{V m}^{-1}$ ), is then defined as the ratio of the force exerted onto  $q$  by the charge  $\Delta q$ , which for the electric field created by a charge  $q$  in vacuum yields

$$\bar{E} = \frac{q}{4\pi\epsilon_0 r^2} \bar{a}_r \quad \text{V m}^{-1} \quad (1.3)$$

Ideally, the electric field is defined in the limit that  $\Delta q$  tends to zero. It is a vector field, radial in the case of a point charge. It comes out of a positive charge and points toward a negative charge. The lines of electric field are tangential to the electric field in every point. Equation (1.3) is linear with respect to the charge. Hence, when several charges are present, one may vectorially add up the electric fields due to each charge, which yields what is often called the generalized Coulomb's law.

The electric charge may appear in four different forms:

1. It can be punctual, as in Eqn. (1.2). It is then usually denoted  $q$  and measured in *coulombs*.
2. It can be distributed in space along a line (material of not). It is then usually denoted  $\rho_l$  and measured in *coulombs per meter* ( $\text{C m}^{-1}$ ).
3. It can be distributed in space over a surface (material of not). It is then usually denoted  $\rho_s$  and measured in *coulombs per square meter* ( $\text{C m}^{-2}$ ).
4. It can also be distributed in a volume. It is then usually denoted  $\rho$  and measured in *coulombs per cubic meter* ( $\text{C m}^{-3}$ ).

When a material is submitted to an applied electric field, it becomes polarized, the amount of which is called the *polarization vector*  $\bar{P}$ . This is due to the fact that, in many circumstances, electric dipoles are created or transformed into the material, which corresponds to what is called the dielectric properties of the material. Hence, the polarization is the *electric dipole moment per unit volume*, in *coulombs per square meter*.

The total electric field in a dielectric material is the sum of the applied electric field and of an induced electric field, resulting from the polarization of the material. As a simple example, a *perfect electric conductor* is defined as an equipotential material. If the points in the material are at the same electric potential, then the electric field must be zero and there can be no electric charges in the material. When a perfect electric conductor is submitted to an applied field, this applied field exists in all points of the material. To have a vanishing *total electric field*, the material must develop an induced electric field such that the sum of the applied field and the induced field vanishes in all points of the material. The induced field is calculated by taking into account the geometry of the problem and the boundary conditions, which can of course be complicated. As another example, a human body placed in an applied electric field develops an induced electric field such that the sum of the applied field and the induced field satisfies the boundary conditions at the surface of the body. The total field in the body is the sum of the applied field and of the induced field.

A new vector field  $\bar{D}$  is then defined, known as the *displacement flux density* or the *electric flux density*, in *coulombs per square meter* similarly to the polarization, defined as

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \quad \text{C m}^{-2} \quad (1.4)$$

This definition is totally general, applying to all materials, in particular to all biological materials. It indeed holds for materials in which [3]:

1. The polarization vector has not the same direction as the vector electric field, in which case the material is *anisotropic*.
2. The polarization can be delayed with respect to the variation of electric field, as is the case in *lossy materials*. All physical materials are lossy, so this is a universal property. It is neglected, however, when the losses are reasonably small, which is not always the case in biological tissues.
3. The polarization is not proportional to the electric field, in which case the material is *nonlinear*.

In all other cases, that is, when the material is *isotropic*, *lossless*, and *linear*, the definition (1.4) can be written

$$\bar{D} = \epsilon_0 \epsilon_r \bar{E} = \epsilon \bar{E} \quad (1.5)$$

which combines the applied and induced fields, hence the external source field and the induced polarization, into the definition of  $\epsilon$  ( $\text{F m}^{-1}$ ), permittivity of the material, product of the permittivity of vacuum  $\epsilon_0$  ( $\text{F m}^{-1}$ ) and the relative permittivity  $\epsilon_r$  (dimensionless) of the material. The *electric susceptibility*  $\chi_e$  is related to the relative permittivity by the expression

$$\epsilon_r = 1 + \chi_e \quad (1.6)$$

It should be stressed that the use of permittivity, relative permittivity, and susceptibility is limited to isotropy, losslessness, and linearity, which is far from being always the case, in particular in biological tissues.

*Dielectric polarization* is a rather complicated phenomenon [4]. It may be due to a variety of mechanisms, which can be summarized here only briefly. The simplest materials are gases, especially when they are rarefied. The simplest variety is formed of *nonpolar gases*, in which the molecules have no electric dipole at rest. When an electric field is applied, an electric dipole is induced. This is a simple case for which a simple model can be used for correctly calculating the polarization. The next category is that of *polar gases*, in which an electric dipole does exist at rest. When an external electric field is applied, the dipole orientation is modified; it essentially rotates. For such a polar rarefied gas, which is still a very simple case, the relationship between polarization and applied field is already found to be nonlinear. When the density increases, modeling becomes much more difficult, and classical physics yields wrong models for compact gases, liquids, and of course solids, in

particular conductors, semiconductors, and superconductors. Classical physics almost completely fails when trying to establish quantitative models. It can however yield some very illuminating insight on the phenomena involved with the dielectric character of materials, in particular about the influence of frequency, as will be shown now.

The *dipolar polarization*, resulting from the alignment of the molecule dipolar moment due to an applied field, is a rather slow phenomenon. It is correctly described by a first-order equation, called after Debye [5]: The dipolar polarization reaches its saturation value only after some time, measured by a time constant called *relaxation time*  $\tau$ . The ability to polarize, called the *polarizability*, is measured by the parameter

$$\alpha_d = \frac{\alpha_0}{1 + j\omega\tau} + C \quad (1.7)$$

where constant  $C$  takes into account the nonzero value of the polarizability at infinite frequency. The relative permittivity related to this phenomenon is

$$\epsilon_r = \epsilon'_r - j\epsilon''_r \quad (1.8)$$

where  $N$  is the number of dipoles per unit volume. It should be observed that the permittivity is a complex quantity with real and imaginary parts. If  $\epsilon_{r0}$  and  $\epsilon_{r\infty}$  are the values of the real part of the relative permittivity at frequencies zero and infinity, respectively, one can easily verify that the equations can be written as

$$\epsilon'_r = \frac{\epsilon_{r0} - \epsilon_{r\infty}}{1 + \omega^2\tau^2} + \epsilon_{r\infty} \quad \epsilon''_r = \frac{(\epsilon_{r0} - \epsilon_{r\infty})\omega\tau}{1 + \omega^2\tau^2} \quad (1.9)$$

The parameter  $\epsilon_{r\infty}$  is in most cases the value at optical frequencies. It is often called the *optical dielectric constant*.

Dipolar polarization is dominant in the case of water, much present on earth and an essential element of living systems. The relative permittivity of water at 0°C is

$$\epsilon_r = 5 + \frac{83}{1 + j0.113f(\text{GHz})} \quad (1.10)$$

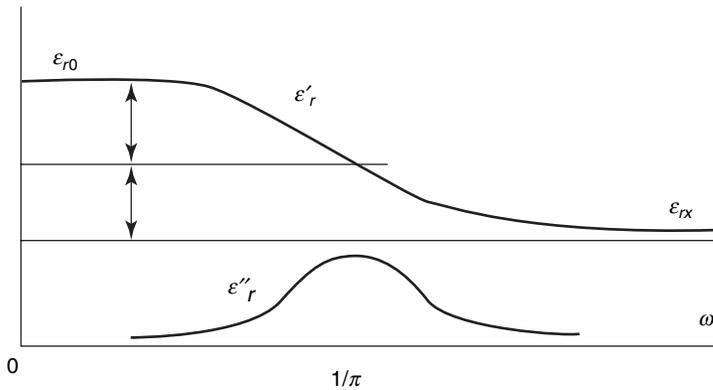
with  $1/\tau = 8.84$  GHz. The real part of the relative permittivity is usually called the *dielectric constant*, while the imaginary part is a measure of the dielectric losses. These are often expressed also as the tangent of the *loss angle*:

$$\tan \delta_e = \frac{\epsilon''}{\epsilon'} \quad (1.11)$$

Table 1.1 shows values of relaxation times for several materials. A high value of the relaxation time is indicative of a good insulator, while small values are typical of good conductors.

**TABLE 1.1 Relaxation Time of Some Materials**

Material	Relaxation Time
Copper	$1.51^{-19}$ s
Silver	$1.31^{-19}$ s
Sea water	$2.01^{-10}$ s
Distilled water	$10^{-6}$ days
Quartz	10 days



**FIGURE 1.1** Relaxation effect.

Figure 1.1 represents the typical evolution of the real and imaginary parts of a relative permittivity satisfying Debye’s law, where  $\epsilon_{r0}$  and  $\epsilon_{r\infty}$  are the values at frequencies zero and infinity, respectively. It shows the general behavior of the real and imaginary parts of permittivity: The imaginary part is nonzero only when the real part varies as a function of frequency. Furthermore, each part can be calculated from the variation of the other part over the whole frequency range, as indicated by the Kramer and Kronig formulas [6]:

$$\begin{aligned} \epsilon'(\omega) &= \epsilon_0 + \frac{2}{\pi} \int_0^\infty \frac{x\epsilon''(x)}{x^2 - \omega^2} dx \\ \epsilon''(\omega) &= -\frac{2\omega}{\pi} \int_0^\infty \frac{\epsilon'(x) - \epsilon_\infty}{x^2 - \omega^2} dx \end{aligned} \tag{1.12}$$

It can easily be seen that  $\epsilon'' = 0$  if  $\epsilon'$  is frequency independent. The variable of integration  $x$  is real. The principal parts of the integrals are to be taken in the event of singularities of the integrands. The second equation implies that  $\epsilon''(\infty) = 0$ . The evaluation of Eqn. (1.12) is laborious if the complex  $\epsilon(\omega)$  is not a convenient analytical function. It is interesting to observe that the formulas are

similar to those relating the real and imaginary parts of impedance in general circuit theory [7].

The structure of Maxwell's equations shows that permittivity and conductivity are related parameters. To keep it simple, one may say that they express the link between current density and electric field: When both parameters are real, the permittivity is the imaginary part while the conductivity is the real part of this relationship. This can be written as

$$\bar{J} = (j\omega\epsilon + \sigma)\bar{E} \quad \text{A m}^{-2} \quad (1.13)$$

When the permittivity is written as complex, there is an ambiguity. There are, however, too many parameters, as can be seen in the expression

$$\begin{aligned} \bar{J} &= j\omega(\epsilon' - \epsilon'')\bar{E} + \sigma\bar{E} \\ &= j\omega\epsilon'\bar{E} + (\sigma + \omega\epsilon'')\bar{E} \end{aligned} \quad (1.14)$$

from which it appears that the real part of the relation between the current and the electric field can be written either as an effective conductivity equal to

$$\sigma_{\text{eff}} = \sigma' + \omega\epsilon'' \quad \text{S m}^{-1} \quad (1.15)$$

or as an effective imaginary part of permittivity equal to

$$\sigma''_{\text{eff}} = \frac{\sigma}{\omega} \quad \text{F m}^{-1} \quad (1.16)$$

It should be observed that these two expressions are for the conductivity and permittivity, respectively, and not just the relative ones. Both expressions are correct and in use. Generally, however, the effective conductivity is used when characterizing a lossy conductor, while the effective imaginary part of the permittivity is used when characterizing a lossy dielectric. At some frequency, the two terms are equal, in particular in biological media. As an example, the frequency at which the two terms  $\sigma'$  and  $\omega\epsilon''$  are equal is in the optical range for copper, about 1 GHz for sea water, 100 MHz for silicon, and 1 MHz for a humid soil. Although both expressions are correct, one needs to be careful in such a case when interpreting the results of an investigation.

One more comment, however, is necessary. It has just been said, and it is proven by Kramer and Kronig's formulas (1.12), that, if the permittivity varies as a function of frequency, it must be a complex function. In fact, this is true of all three electromagnetic parameters: permittivity, conductivity, and permeability. Hence, by writing the conductivity as a real parameter in Eqns. (1.13)–(1.16), we have assumed that it was independent of frequency, which is about the case of the steady conductivity. If it varies with frequency, then it has to be written as a complex parameter  $\sigma' = \sigma' + j\sigma''$  and Eqns. (1.13)–(1.16) have to be modified consequently.

*Ionic polarization* and *electronic polarization* are due to the displacement of the electronic orbits with respect to the protons when an electric field is applied. This phenomenon is much faster than dipolar polarization. It is a movement and is described adequately by a second-order equation, characterized by possible resonance [3, 5]. Dielectric losses induce damping, which makes the permittivity complex in the frequency domain, as for the first-order effect described by Eqn. (1.6). The real and imaginary parts of the relative permittivity are classical expressions for a second-order equation:

$$\begin{aligned} \epsilon'_r &= 1 + \frac{q^2 N}{\epsilon_0 m} \frac{\omega'^2 - \omega^2}{(\omega'^2 - \omega^2)^2 + \gamma^2 \omega^2} \\ \epsilon''_r &= \frac{q^2 N}{\epsilon_0 m} \frac{\gamma \omega}{(\omega'^2 - \omega^2)^2 + \gamma^2 \omega^2} \end{aligned} \tag{1.17}$$

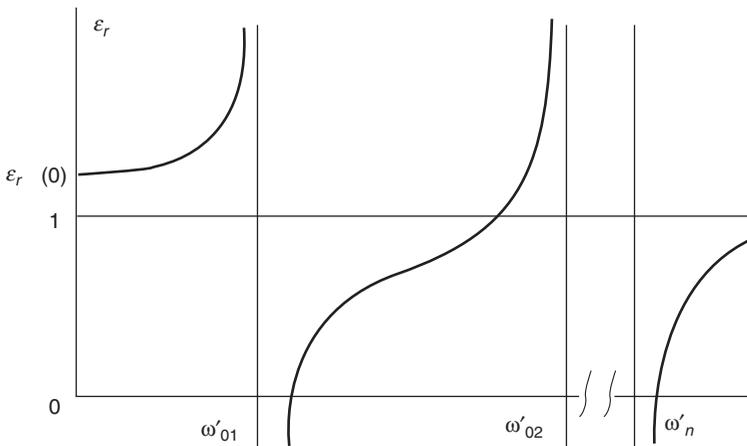
with

$$\omega_0'^2 = \omega_0^2 - \frac{q^2 N}{\epsilon_0 m} \tag{1.18}$$

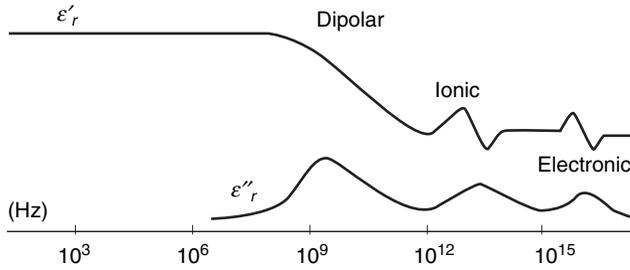
where  $m$  is the moving mass,  $\gamma$  the damping term,  $-q$  the moving charge, and  $N$  the number of active elements per unit volume. When the damping term vanishes, the permittivity is purely real:

$$\epsilon_r = 1 + \frac{q^2 N}{\epsilon_0 m} \frac{1}{\omega'^2 - \omega^2} \tag{1.19}$$

In fact, the oscillation frequencies  $\omega_0'^2$  are not identical for all the electrons in a given volume, which requires a generalization by adequately summing all the contributions. Figure 1.2 represents the typical evolution of the real and



**FIGURE 1.2** Ionic and electronic polarization (from [3], courtesy of De Boeck, Brussels).



**FIGURE 1.3** Permittivity as a function of frequency (from [3], courtesy of De Boeck, Brussels).

imaginary parts of a relative permittivity satisfying the second-order equation of movement. When the damping term decreases, the variation of the real part of the permittivity becomes steeper while the maximum of the imaginary part increases and tends to infinity at the oscillation frequency.

When submitted to electric fields, an actual material exhibits a variety of relaxation and resonance phenomena. Figure 1.3 represents a typical variation of the real and imaginary parts of the relative permittivity as a function of frequency.

Some materials exhibit unusual properties when submitted to electric fields. For instance, barium titanate and some other materials are *ferroelectric*: The electric polarization does not vary linearly with the amplitude of the applied field, exhibits hysteresis, and varies significantly with temperature. A critical temperature, above which ferroelectricity disappears and termed the *Curie temperature*, is associated with the phenomenon. Some other materials are called *antiferroelectric*: Their electric dipoles are very much organized as well; they are antiparallel, however.

### 1.3.2 Magnetic Field and Flux Density

The magnetic field  $\vec{H}$  is derived from Ampere's law, which expresses the interaction between two loops of wires carrying currents. The total force experienced by a loop is the vector sum of forces experienced by the infinitesimal current elements comprising the second loop. Experimentally, it has been shown that [3, 4–7]

1. Two currents of equal directions attract each other, while they repel when they have opposite directions.
2. The magnitude of the force is proportional to the product of the two currents and to the product of the lengths of the two current elements.
3. The magnitude of the force is inversely proportional to the square of the distance between the current elements.

4. The force acts along the cross product of one current element by the cross product of the other current element by the unit vector along the line joining the two current elements.
5. The force depends upon the magnetic properties of the medium in which these are placed.

It is usual to calculate directly the contribution to the magnetic field evaluated at an observation point  $P$  of a current element  $I'$ , expressed in *amperes* (A), directed along

$$\overline{dH}(\vec{r}) = \frac{I'(\vec{r}')d\vec{l}'x(\vec{r} - \vec{r}')}{4\pi|\vec{r} - \vec{r}'|^3} \quad \text{A m}^{-1} \quad (1.20)$$

where the factor  $4\pi$  is due to the use of the rationalized MKS system. The magnetic field is a vector field, circumferential in the case of a linear current. The lines of magnetic field are tangential to the magnetic field in every point. Equation (1.20) is linear with respect to the current.

This law shows similarities with Coulomb's law,

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \vec{a}_r, \quad [\text{Eq. (1.3)}]$$

since, for a current element located at the origin of the coordinates ( $\vec{r}' = 0$ ), the modulus of the magnetic field equals

$$|\overline{dH}| = \frac{I dl'}{4\pi r^2} \sin(\overline{dl}', \vec{r}) \quad (1.21)$$

The total value of the magnetic field at point  $P$  is obtained by integrating Eqn. (1.14) over all the possible currents:

$$\vec{H}(\vec{r}) = \int_{\text{source}} \frac{I'(\vec{r}')d\vec{l}'x(\vec{r} - \vec{r}')}{4\pi|\vec{r} - \vec{r}'|^3} \quad (1.22)$$

The electric current may appear in three forms:

1. It can be a current element as in Eqn. (1.20). It is then usually denoted  $\vec{I}$  and expressed in *amperes*.
2. It can be a surface current density. It is then usually denoted  $\vec{K}$  and expressed in *amperes per meter* ( $\text{A m}^{-1}$ ).
3. It can be a current density. It is then usually denoted  $\vec{J}$  and expressed in *amperes per square meter* ( $\text{A m}^{-2}$ ).

When a material is placed in an applied magnetic field, it becomes magnetized, the amount of which is called the *magnetization vector*  $\vec{M}$ . This corresponds to what is called the magnetic properties of the material, which can be due to a variety of effects. In particular, as for the electric polarization, it may be due to the fact that magnetic dipoles are created or transformed into

the material. In this case, the magnetization is the *magnetic dipole moment per unit volume*, in *amperes per square meter*.

The total magnetic field in a material is the sum of the applied magnetic field and an induced magnetic field, resulting from the magnetization of the material. Earlier in this section, the perfect electric conductor has been defined as an equipotential material in which all the points are at the same electric potential. In such a material, the total electric field must be zero, which implies that the material must develop an induced electric field such that the sum of the applied field and the induced field vanishes in all points of the material. On the other hand, one may define the *perfect magnetic conductor* as the dual of the perfect electric conductor, in which all the points are at the same magnetic potential. In such material, the total magnetic field must be zero, which implies that the material must develop an induced magnetic field such that the sum of the applied field and the induced field vanishes in all points of the material. The definition of the perfect electric conductor is very useful, largely because there are in nature materials whose properties are very close to those of the perfect electric conductor, for example, gold, silver, and copper. There is, however, no physical material with properties very close to those of the perfect magnetic conductor.

The induced magnetic field is calculated by taking into account the geometry of the problem and the boundary conditions, which can of course be complicated. The human body is essentially nonmagnetic, hence transparent to magnetic fields, except however for some localized magnetic properties.

A new vector field  $\bar{B}$  is then defined, known as the *magnetic flux density*, in *webers per square meter*, similarly to the magnetization, or *tesla*, defined as

$$\bar{B} = \mu_0(\bar{H} + \bar{M}) \quad \text{Wb m}^{-2}, \text{ T} \quad (1.23)$$

This definition is totally general, applying to all materials. It indeed holds for materials in which [3]:

1. The magnetization vector has not the same direction as the vector magnetic field, in which case the material is *anisotropic*.
2. The magnetization can be delayed with respect to the variation of magnetic field. This is the case in all *lossy materials*, and all materials are lossy, so this is a universal property. It is neglected, however, when the losses are reasonably small.
3. The magnetization is not proportional to the magnetic field, in which case the material is *nonlinear*.

In all other cases, that is, when the material is *isotropic*, *lossless*, and *linear*, the definition (1.23) can be written

$$\bar{B} = \mu\bar{H} \quad (1.24)$$

which combines the applied and induced fields, hence the external source field and the induced magnetization. The *permeability* of the material is defined by

$$\mu = \mu_0(1 + \chi_d) = \mu_0\mu_r \quad \text{H m}^{-1} \quad (1.25)$$

It is the product of the permeability of vacuum  $\mu_0$  (in henrys per meter) by the relative permeability  $\mu_r$  (dimensionless) of the material. Equation (1.25) defines at the same time the *magnetic susceptibility*  $\chi_d$ . The use of permeability, relative permeability, and magnetic susceptibility is limited to isotropy, losslessness, and linearity, which is far from being always the case. It should be stressed again, however, that biological tissues are essentially nonmagnetic. Magnetization is a very complicated phenomenon [3, 4–7]. It may be due to a variety of mechanisms, which can be summarized here only briefly.

All materials are diamagnetic: *Diamagnetism* is a general property of matter. It is due to the fact that electrons placed in a magnetic field have their rotation speed increased, which results in an induced magnetic moment opposed to the applied magnetic field, hence in a decrease of magnetization. The diamagnetic susceptibility is negative and very small, normally of the order of  $-10^{-8}$  to  $-10^{-5}$ . It does not depend explicitly upon the temperature; it depends however upon the density of the material. A number of metals used in engineering are only diamagnetic, for instance, copper, zinc, gold, silver, cadmium, mercury, and lead. As an example, the relative permeability of copper is 0.999991.

When the atoms of a material have a permanent magnetic moment, this moment tends to align itself with the applied magnetic field to minimize the magnetic energy. This phenomenon is similar to electric polarization. It is a rather slow phenomenon, correctly described by a first-order law. It characterizes *paramagnetism*, in which the magnetization is positive: It reinforces the effect of the applied field. It should be noted that paramagnetic materials are also diamagnetic. Diamagnetism, however, is much smaller than paramagnetism and is not observable in this case. Paramagnetic magnetization is modeled similarly to electric polarization. It depends upon temperature and density. The paramagnetic susceptibility is positive and, in general, much smaller than 1. Some metals are paramagnetic, for example, aluminum, platinum, manganese, magnesium, and chromium. As for electric polarization, when the material density increases, modeling becomes much more difficult, and classical physics yields wrong models. Classical physics almost completely fails when trying to establish quantitative models. It can, however, yield some very illuminating insight on the phenomena involved with the dielectric character of materials, in particular about the influence of frequency.

The relative permeability related to this phenomenon is

$$\mu_r = \mu'_r - j\mu''_r \quad (1.26)$$

It is a complex quantity, with real and imaginary parts. The imaginary part is a measure of the magnetic losses. These are often expressed also as the tangent of the *loss angle*:

$$\tan \delta_m = \frac{\mu''}{\mu'} \quad (1.27)$$

As for the electric polarization, the imaginary part of the permeability is nonzero only when the real part varies as a function of frequency. Furthermore, each part can be calculated from the variation of the other part over the whole frequency range.

In certain materials, the application of a magnetic field may induce an extremely strong magnetization, of the order of  $10^8 \text{ A m}^{-1}$ , which is 10 times more than the highest fields which can be produced in laboratory. This characterizes the phenomena that are called *ferromagnetism*, *ferrimagnetism*, *antiferromagnetism*, and *antiferromagnetism*. An even simple explanation of the underlying physics is far beyond the scope of this book. Classical physics indeed fails in trying to model these phenomena. In fact, quantitative modeling of ferromagnetism was the reason Dirac developed quantum mechanics. The phenomenon is based on an exchange probability for some electrons between neighboring atoms in the crystalline structure, which corresponds to a spin coupling. The interaction from neighbor to neighbor is positive if the spins are parallel, which characterizes ferromagnetism; it is negative if the spins are antiparallel, which characterizes antiferromagnetism. The conditions for the exchange to take place are the following:

1. The distance between the electron and nucleus may be neither too large nor too small: it must be of the order of 0.27–0.30 nm; the materials in which this distance is smaller than 0.26–0.27 nm are practically antiferromagnetic.
2. The atoms must have nonsaturated layers to be able to accept electrons.
3. The temperature has to be lower than a critical temperature, called the *Curie temperature*, to avoid too much thermal disorder; above this temperature, the materials become paramagnetic.

Conditions 1 and 2 imply the material has a nonsaturated layer at the right distance from the nucleus: It is the third layer, and it is nonsaturated in the case of iron, nickel, and cobalt, which are the ferromagnetic materials.

Ferrites are ferrimagnetic materials. They are ceramics, generally composed of oxides of iron and other metals. Their chemical formula is very often  $ZO \cdot Fe_2O_3$ , where Z represents one or more divalent metals (cobalt, copper, manganese, nickel, and zinc). In most common ferrites, Z is for the combinations Mn + Zn or Ni + Zn. The size of the grains in ferrites is of the order of 1–20  $\mu\text{m}$ . The oxygen anions play an important role in ferrimagnetics: Their presence results in a magnetization much smaller than that of ferromagnetic materials, because the magnetic cations are distributed in a nonmagnetic oxygen network. Furthermore, because of the oxygen ions, the alignment of the cation moments is antiparallel in some places of the network. When the antiparallel subnetworks have magnetic moments equal per unit volume, the material is antiferromagnetic.

Most ferrites are polycrystalline. They usually have a high permeability, relatively small losses, and a high electrical resistivity. The saturation magnetic flux density is lower than in ferromagnetic materials: It is of the order of 0.5 T in a MnZn ferrite instead of 2 T in iron.

Several types of energy have to be taken into account in investigating ferro- and ferrimagnetic materials:

1. Exchange energy, corresponding to the energy difference between the elements with parallel spins and those with antiparallel spins.
2. Magnetostatic energy, associated with free magnetic poles in the material.
3. Magnetostriction energy, related to the fact that a magnetic material undergoes an elastic deformation in the direction of magnetization, which may be positive or negative, corresponding to an elongation or a contraction, respectively—this deformation is of the order of a few millionths.
4. Anisotropic energy of the crystal, associated with the difference between the direction of magnetization and those of easy magnetization.

### 1.3.3 Electromagnetic Field

In DC situations, the electric field is calculated correctly from the laws of electrostatics. The same is true at extremely low and low frequencies, although the approximation is less applicable when the frequency increases. Similarly, the magnetic field is calculated from the laws of magnetostatics in DC situations. The same is true at extremely low and low frequencies, with however a quality of approximation decreasing when the frequency increases. Values of the frequency at which the approximation is not valid anymore depend on the geometric, electrical, and magnetic properties of the problem.

When the frequency increases, the electric and magnetic fields cannot be separated from each other: If one of the fields exists, so does the other. One cannot consider one field as the source of the other. They are linked to each other in every situation, and this is described by *Maxwell's equations*. Maxwell, a British physicist, formalized the laws of electromagnetics around 1880, without writing the equations however as we know them today. He made at this occasion two essential contributions:

1. One was to say that all the former laws, essentially based on experimental measurements made by Gauss, Ampere, Lenz, and others, were valid but that they had to be considered as a system of equations.
2. The other was to point out that, in the ensemble of these laws, one term was missing: The displacement current did not appear in the former law and it was to be added.

A consequence of the first contribution is that the electric and magnetic fields are linked together; hence there is a coupling between electricity and magnetism. As a consequence of the second contribution, the electric current can take one more form than previously known, so that one could have the following (the electric current passing through a given area is obtained by surface integrating the corresponding current density):

1. A *convection current density*, due for instance to a density of electric charges moving in vacuum, is described by

$$\bar{J}_{\text{conv}} = \rho \bar{v} \quad (1.28)$$

2. A *conduction current density*, due to the conductivity of some materials, is described by a relationship based on the electric current in the conductor:

$$\bar{J}_{\text{cond}} = \sigma \bar{E} \quad (1.29)$$

3. A *displacement current density*, due to the time variation of the electric field, is equal to the time derivative of the displacement field (the electric current density):

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} \quad (1.30)$$

If we consider an alternate-current source feeding a capacitor formed of two parallel plates in vacuum through an electric wire, the displacement current ensures the continuity between the conduction current circulating in the wire and the electric phenomenon involved between the two capacitor plates: The displacement current density, integrated over the area of the plate and time derived, that is, multiplied by  $j\omega$ , is equal to the conduction current in the wire [3, 7].

4. A *source current density*.

Maxwell's equations form a system of first-order equations, vector and scalar. They are usually considered as the generalization of the former laws describing electricity and magnetism. This is a historical point of view, usually accompanied by the comment that up to now no EM phenomenon has been pointed out which does not satisfy Maxwell's equations. It has been demonstrated, however, that Maxwell's equations can be derived from a relativistic transformation of Coulomb law, under the constraint of the speed of light constant with respect to the observer. The demonstration is not too difficult in the case of linear motion, because then special relativity may be used [1–3]. It involves, however, a significant amount of vector calculus. In the case of rotation, however, general relativity must be used, which makes the

calculation much more complicated [8]. As a result, it can be concluded that if an EM phenomenon is found which does not satisfy Maxwell's equations, Coulomb's law, relativity, or the constancy of the speed of light has to be revisited.

Maxwell's equations are not valid at boundaries, where *boundary conditions* have to be used. They may be written in either the time or frequency domain. They may also be written in either differential or integral form. As a consequence, they can take four different forms [3]. The *differential* forms, in the time or frequency domain, are used to describe the EM phenomena in continuous media. They are punctual and valid in every point of a continuous medium submitted to EM fields. The *integral* forms, on the other hand, link average quantities; they are applicable to volumes, surfaces, and contour lines. They are generally used to investigate circuits, mobile or not. *Time-domain* descriptions evaluate real, physically measurable quantities. They must be used when the physical consequence is not in synchronism with the cause, for instance, when investigating some optoelectronic phenomena. They must also be used when investigating wide-band phenomena. They are valid at any frequency. *Frequency-domain* descriptions are extremely useful when investigating monochromatic or narrow-band phenomena. The quantities they evaluate are complex numbers or vectors: They are not real, physically measurable quantities, although they may have a real physical content. They are valid at any time.

Solving Maxwell's equations in general cases may be quite a difficult task. Decomposing the vector equations into scalar ones, the system to be solved has a total of 16 equations. There are also 16 unknowns: the 5 vector quantities of electric field, electric flux density, magnetic field, magnetic flux density, and current density, that is, 15 scalar quantities, and the scalar charge density. Fortunately, in many cases, the system is much simpler, for instance because of symmetry, simplicity of the materials involved, slowly varying phenomena, and low frequency. A fair amount of symmetry, in particular, may greatly simplify calculations.

In living tissues, electromagnetic phenomena are usually slow, when compared to the extremely broad variety of phenomena to be evaluated in physics and engineering. The shortest biological response time indeed is of the order of  $10^{-4}$  s, while most biological reactions are much slower. Hence, Maxwell's equations are most generally not used for evaluating biological effects in living tissues and systems. Furthermore, in this book we are interested in RF and microwave stimulation. At RFs, the wavelength in vacuum is large with respect to living tissues and systems, including human beings as a whole: At 1 MHz, the wavelength in vacuum is equal to 300 m. On the other hand, at microwaves, the period of oscillation is small, equal to  $10^{-9}$  s at 1 GHz, which is much smaller than the fastest biological responses. This implies that the evaluation of biological effects in living tissues and systems due to EM stimulation, at RF as well at microwaves, in most cases does not necessitate the use of Maxwell's equations. Hence, in practice, quasi-static approaches are quite satisfactory in

biological material, and the electric and magnetic fields are very often considered separately, even at RF/microwaves.

### 1.3.4 Electromagnetic Wave

Investigating the structure of Maxwell's equations, Hertz found, around 1888, both theoretically and experimentally, that it included the notion of propagation of EM waves because of the specific coupling between the electric and magnetic fields due to the particular form of the vector equations: The coupling between space variations and time variations of the electric and magnetic fields results in wave motion. The waves were propagating at a finite speed, the "speed of light." A few years later, in 1895, Marconi starting experimenting this at larger and larger distances, establishing what was called *wireless communications*.

Propagation has a precise mathematical definition: All the components of fields and associated physical quantities, such as current and charge densities, have a  $z$  dependence expressed as the factor  $e^{-jkz}$  in a cylindrical coordinate system or an  $r$  dependence expressed as the factor  $e^{-jkr}$  in a spherical coordinate system. Such an ensemble of fields is called an EM wave. Hence, the words *propagation* and *wave* are closely related [9].

Maxwell's equations are first-order equations. Eliminating one of the fields in these equations yields a second-order equation for the other field, which is called the *wave equation* or *Helmholtz equation*. The general solution of the wave equation is a propagating EM wave. Even in a medium as simple as vacuum, there are a variety of waves satisfying the wave equation, with transverse and/or longitudinal components of the fields. The field structure of the wave also depends on the fact that the propagation medium is either infinite or bounded: A coaxial cable and a waveguide have lateral bounds. When the medium is unbounded, it is often called *free space*. When the medium is bounded, *reflection* occurs on the interfaces. When edges or corners are present, *refraction* may occur.

On the other hand, the orientation of the fields may vary in space when the wave is propagating, depending on the type of wave and the medium. What characterizes this variation is called the *polarization*. It is related most often to the electric field, although it may also be related to the magnetic field. In this case, however, it should be mentioned explicitly.

The simplest EM wave structure is well known from elementary physics. It has a magnetic field perpendicular in space to the electric field, both being perpendicular in space to the direction of propagation. The fields are stationary neither in space nor in time: The wave displaces itself in space as a function of time at constant speed and the field amplitudes vary as a function of time, sinusoidal in a monochromatic case. During this variation, when the fields remain parallel to themselves, the polarization is called *linear*. When, on the contrary, the fields rotate at constant amplitude in a plane perpendicular to the direction of propagation, the polarization is called *circular*. Circular polar-

ization may be right handed or left handed. If the amplitude and the direction of the fields both vary when propagating, the polarization is termed *elliptic*. In practice, many RF and microwave sources operate at linear polarization. It should not be assumed, however, that this is always the case: Circular polarization is also used in technical applications.

One way of classifying waves is based on geometric considerations, characterizing the geometry of the surface of constant phase of the wave. Depending on whether this surface is a plane, a cylinder, or a sphere, the wave is *planar*, *cylindrical*, or *spherical*. Furthermore, when the amplitude of the field is the same at every point of the surface of constant phase, which makes the surface of constant phase also a surface of constant amplitude, the wave is *uniform*; otherwise it is *nonuniform*. A *uniform plane wave* is a wave whose surfaces of constant phase are planes on which the amplitude of the field is a constant. As an example, the simplest wave structure described above is a uniform plane wave.

Another way of classifying waves is based on the number of field components. When a wave has only transverse field components for both the electric and magnetic field and no longitudinal components, the wave is *transverse electromagnetic* (TEM). A TEM wave has only four components. When, in addition, the wave has one longitudinal component only, it is called *transverse electric* (TE) if the longitudinal component is magnetic and *transverse magnetic* (TM) if the longitudinal component is electric. Both TE and TM waves have only five components. The most general wave has six components. It is a linear combination of TE and TM waves, possibly together with a TEM wave, depending upon the boundary conditions.

Transverse electromagnetic waves are very much appreciated in practice because they have only four components, with no longitudinal components:  $E_z = 0$  and  $H_z = 0$ . On the other hand, uniform plane waves also characterize a very simple structure. It should, however, be well noted that a TEM wave is not necessarily a uniform plane wave. As an example, the very well known coaxial cable has a TEM wave as the main propagating mode. This TEM wave, however, is not a uniform planar wave. The surface of constant phase is indeed a plane, perpendicular to the direction of propagation, hence the wave is planar. The field, however, is not constant over the plane because it varies between the two conductors according to a  $1/r$  law. Hence, the TEM wave in a coaxial cable is a nonuniform plane wave.

Specific parameters describe EM waves. The product of the *wavelength*  $\lambda$  (in meters) and the frequency is a constant, called the *speed of light* in vacuum, more correctly called the *phase velocity* in the medium where the propagation takes place, for instance inside a human body:

$$f\lambda = c(\text{vacuum}) = v_{\text{ph}}(\text{material}) = \sqrt{\frac{\epsilon}{\mu}} \quad \text{m s}^{-1} \quad (1.31)$$

The speed of light is about  $300,000 \text{ km s}^{-1}$  in vacuum. The last equality is strictly valid for TEM waves only. Being interested essentially in RF and

microwave excitation, we shall find useful to note that the equation can be written as

$$f \text{ (MHz)} \times \lambda \text{ (m)} = 300 \text{ in vacuum} \quad (1.32)$$

Equation (1.31) shows that the wavelength is inversely proportional to the frequency. In vacuum, the wavelength is 6000 km at 50 Hz, while it is 300 m at 1 MHz, 0.3 m at 1 GHz, and 1 mm at 100 GHz. It also shows that the phase velocity decreases when the relative permittivity and/or permeability increases. It is important to observe that the phase velocity may vary significantly from one material to the other, in particular because of the presence of water. Water is a dielectric material with a very high dielectric constant, of the order of 80 at low frequencies, as will be shown later. Most living tissue contains a significant amount of water. As a consequence, the phase velocity at 1 GHz in a human body is almost 9 times smaller than in vacuum because the wavelength is almost 9 times smaller than in vacuum. At higher frequency, however, the permittivity decreases and the values of wavelength and phase velocity are closer to their values in vacuum.

Some parameters are typical of propagation. The *wave number* measures the number of wavelengths per unit length. In physicochemistry, for instance in spectroscopy, it is still rather usual to characterize a frequency by the corresponding wave number. This always looks surprising to a physicist or an engineer, especially when the unit length is a familiar unit rather than the meter. As an example, a wave number of  $1000 \text{ cm}^{-1}$  characterizes a frequency at which there are 1000 wavelengths over a length of 1 cm, hence with a wavelength of  $1 \mu\text{m}$ , that is, in the infrared (IR).

On the other hand, the ratio of electric to magnetic field amplitudes is the *intrinsic impedance* of the propagation medium. In vacuum, it is about 377 (or  $120\pi$ ) ohms. It is also called the *wave impedance*. In the nineteenth century, the impedance of the propagation medium, in particular a vacuum, has long been considered as puzzling. It even led some to imagine the presence of a specific media, the *ether*, with no other characteristics than the permittivity and permeability of a vacuum, offering the wave an adequate medium to propagate. The wave impedance does not correspond to power absorption because the wave can propagate in a lossless medium. This is due to the fact that, even in the simple TEM structure, the electric field is not in phase with the magnetic field: Both are in quadrature, as has been said before. The wave impedance expresses the ability of a propagating wave to transport power density from one point to another in space and have it possibly absorbed there, in total or in part. This will be explained in Section 1.4.1.

Power absorption is a very important concept when investigating biological effects, as will be illustrated in Chapters 1–3. It is also important when designing materials for protecting biological systems in an EM environment, including the medical environment. Recently, EM environments have become very complex because of the wide and rapid spread of many kinds of electric

or electronic devices, as exemplified by recent cellular telephone progress. As a result, EM wave interference problems due to these devices have increased in frequency. Also, biological effects based on these kinds of EM wave radiation have been feared. As a method of countermeasure for protecting biological and medical environment as well as the measurement involved, knowledge of EM wave absorbers has become significant. This will be analyzed in detail in Chapter 5.

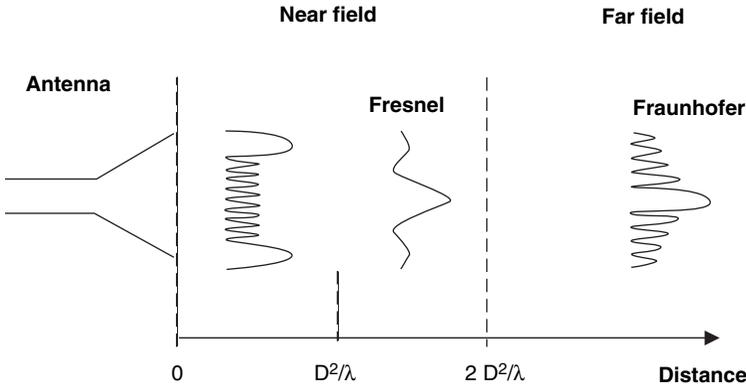
Phase velocity is the velocity at which the phase of a wave propagates. It is also the velocity at which energy propagates, when the medium properties do not vary with frequency. It may not totally characterize the propagation of a wave, however. When the EM properties of the propagating medium vary with frequency, the energy does not propagate at the phase velocity; it propagates at the *group velocity*, which is the frequency derivative of the phase velocity. When the EM properties of the medium vary very significantly with frequency, it may even happen that the *energy velocity* is not the group velocity any more. This is not to be considered in biological tissues and systems, however.

### 1.3.5 Antennas and Near Field

The evaluation of biological effects, including hazards, and medical applications is related to situations where biological tissues and living systems are placed in specific EM environments. In this book, we limit ourselves to RF and microwave environments. Antennas transmit the fields. For instance, they may be placed in free space and have other purposes than illuminating human beings, such as transmitting television, FM radio, or mobile telephony signals. They may also be placed in specific locations, within a part of a human body, for instance, to exert a specific medical effect. In this case, the antennas are often called applicators. On the other hand, fields may have to be measured, for instance in radiometric applications. The antenna being most generally a *reciprocal device*, transmitting and receiving properties are similar. The fields radiated by antennas of finite dimensions are spherical waves. Some applicators are cylindrical and transmit cylindrical waves over the cylindrical portion of the antenna. Because of the importance of the antenna in the exposure situations, some elements about antenna theory have to be reviewed [10].

The space surrounding an antenna in a transmitting or a receiving mode is divided into different regions, usually three: (a) reactive near-field, (b) radiating near-field (Fresnel), and (c) far-field (Fraunhofer) regions (Fig. 1.4). The field structure is different in each region, without abrupt changes, however, from one region to the neighboring one.

The *reactive near-field region* is the region immediately surrounding the antenna, where the reactive field predominates. For most antennas, this region is commonly taken as interior to a distance  $0.62(D^3/\lambda)^{1/2}$  from the antenna surface, where  $\lambda$  is the wavelength and  $D$  the largest dimension of the antenna. The region is called reactive because the reactive power density predominates in this region.



**FIGURE 1.4** Angular distribution of fields around an antenna in the zone of near fields (Fresnel zone) and far fields (Fraunhofer zone).

The *radiating near-field (Fresnel) region* extends from the reactive near-field limit to a distance  $2D^2/\lambda$ , where  $D$  is the largest dimension of the antenna. For this expression to be valid,  $D$  must also be large compared to the wavelength. If the antenna has a maximum overall dimension which is very small compared to the wavelength, this field region may not exist. For an antenna focused at infinity, the radiating near field is sometimes referred to as the Fresnel region on the basis of analogy to optical terminology. In this region the field pattern is, in general, a function of the radial distance and the radial field component may be appreciable.

The *far-field (Fraunhofer) region* is commonly taken to exist at distances greater than  $2D^2/\lambda$  from the antenna. This criterion is based on a maximum phase error of  $\pi/8$ . The reference to Fraunhofer is due to analogy to optical terminology, when the antenna is focused at infinity. In this region the fields are essentially transverse and the angular distribution is independent of the radial distance where the measurements are made.

In RF and microwave communications, far-field situations are usually the ones of most practical concern. This is very convenient because approximations can then be made to obtain closed-form solutions for the fields. This is not always the case when evaluating biological effects, and it is very important to clearly distinguish between near-field and far-field exposure. The evaluation of *hazards* due to RF/microwave exposure on human beings or animals is usually made in far-field conditions. Transmitting stations are normally far enough from living and working situations on the one hand while the antenna of a mobile telephone is so small with respect to the wavelength that the head of an end user is in the far field of the antenna on the other hand. The evaluation of *specific biological effects*, however, in particular in medical applications, is usually made in near-field conditions, as will be obvious later in this

book. In such cases, near-field calculations have to be done, which may require much more attention.

As has been said before, antennas are reciprocal, except for very special devices. Hence, near-field situations have to be taken into account not only for transmitting antennas such as TV and FM radio transmitters but also for receiving antennas. Antennas can indeed be implanted in living tissues and organisms for medical applications.

When the antenna is used to deliver microwave power to heat tissue, the size and location of the microwave field have to be carefully located to control the affected tissue. Hence, the type and shape of the antenna are very much dependent upon the specific application, and there are a variety of applicators. A main problem is of course that of matching the applicator to the tissue. However, EM energy transfer depends, to a great extent, on the absorption properties of the tissue. It also depends on the frequency. As an example, sources at millimeter waves yield results similar to IR frequencies. These aspects will be investigated in detail in Section 1.5.

## 1.4 RF AND MICROWAVE ENERGY

### 1.4.1 Power and Energy

Adequately combining Maxwell's equations yields what is called *Poynting's theorem*. In the time domain, it expresses *equality between the spatial variation of EM power and the time variation of EM energy*, the sum of the electric and magnetic energies [3, 6, 7, 11]. The cross product of the electric field and the magnetic field is called the Poynting vector, with units of volt-amperes per square meter (watts per square meter). Being a cross product, it is perpendicular to the plane of the two vectors. In Section 1.3.4, we have seen that the TEM wave is the simplest wave structure, with the electric field perpendicular to the magnetic field, both fields being perpendicular to the direction of propagation. Hence, the Poynting vector of a TEM wave is in the direction of propagation.

Power and energy have not to be confused. Electromagnetic power is represented by the Poynting vector, just described. The integration of this vector over an open surface yields the power flow through the surface, in *watts*. The integration over a closed surface, with the normal to the surface considered as positive when extending outside the surface, also yields the power flow through the surface, that is, the total power coming out of the volume bounded by the closed surface. If this power is negative, it means that the net power is entering the volume, which also means that the medium inside the volume has absorbing losses—electric, magnetic, or conductive. The total power absorption is obtained by integrating the losses over the whole volume: *Absorption is associated with power*.

On the other hand, in the domestic sense of the word, energy means power absorption for some time in *watt-seconds*. In electromagnetics, however,

energy is interpreted as a field concentration, stored in space, expressed in *joules* (J). The total electromagnetic energy stored in a given volume is obtained by integrating the energy over the volume. The time derivative of energy yields watts, hence power.

We should now have a better understanding of the second sentence of this section: In the time domain, Poynting's theorem expresses an equality between the spatial variation of EM power and the time variation of EM energy, the sum of the electric and magnetic energies. It is often said that Poynting's theorem expresses the conservation of energy. What it precisely expresses is that, for a given volume, if there is a net flow of EM power penetrating into the volume, then the EM energy increases in the volume, a possible difference between the two quantities being the power dissipation within the volume because of the medium conductivity. In the time domain, Poynting's theorem can be expressed in either differential or integral form.

Expressed in the frequency domain, the real part of the complex Poynting vector at a point is equal to the average value of the real power flux, physically measurable, at that point. When integrated over the surface limiting a given volume, it is equal to the real power dissipated in the considered volume due to whole of the electric, magnetic, and conductive losses. Contrary to the time-domain theorem, the frequency-domain theorem shows that the imaginary part of the Poynting vector is not related to the total frequency-domain EM energy: It is related to the difference of the magnetic and electric energies. Hence, it vanishes when the two energies are equal. This situation is called *resonance*, where the power flux is entirely real. In the frequency domain also, Poynting's theorem can be expressed in either differential or integral form.

Poynting's theorem can be used in establishing a general expression for the impedance of an EM structure, for instance an antenna [3]. The structure is placed inside a virtual closed surface and the expression relates the energy stored and the power dissipated in the bounded volume.

Poynting's theorem expresses the equality between the space variation of the EM power and the time variation of the EM energy. This form of the theorem is sufficient in most cases, at least in media where the current is a conduction current. In some cases, however, a generalized form may be necessary, for instance when the current is a convection current, due to moving charges, in vacuum or other media. This may be the case in plasmas, magnetohydrodynamics, and microwave tubes. Tonks has established such a generalized form of Poynting's theorem [12], obtaining equality for the conservation of energy, where the power is the sum of EM term and a kinetic term, while the energy is the sum of EM energy and kinetic energy. On both sides of the equations, the EM and kinetic terms cannot be separated. This expresses the possibility of transforming one term into the other, for instance EM energy into kinetic power, as in a particle accelerator, as well as kinetic energy into EM power, as in solar eruptions. In most cases, however, and in biological applications in particular, the usual form of Poynting's theorem is quite satisfactory.

Some special care has to be exerted when the EM properties of a medium vary with frequency [6]. The medium is then said to be *dispersive*. It has been shown that such a material is necessarily *absorptive*. The fundamental problem is that, in this case, EM energy has no precise thermodynamic definition. When the medium has limited dispersion, it is said to be *transparent*. This is the case when permittivity and permeability vary only slowly around the operating frequency. The mean value of the total EM energy can then be calculated and used in Poynting's theorem. Such a calculation might be necessary in biological tissues and systems, but only when operating in a range of frequencies where permittivity or permeability varies with frequency. It should be remembered that, in this case, the wave energy does not propagate at the phase velocity; it propagates at the group velocity, or even at another velocity if the dispersion is high, as has been mentioned in Section 1.3.4.

Finally, the pattern of energy absorption in biological tissues or systems, as in the human body, contributes to the RF/microwave effect. This raises the question of whether a whole-body average absorption rate can be used as the only determining factor in evaluating biological effects of RF and microwaves. Other features of the radiation also need to be considered. This will be considered in Chapter 2. A radiation diagram typical of a communications antenna is shown in Figure 1.5: A paraboloid antenna is placed at the coordinate origin. As illustrated, the gain  $G_i(\theta, \phi)$  of the antenna varies with the direction  $(\theta, \phi)$ . The *gain* of an antenna is defined as the ratio of the power transmitted by the antenna in a given direction to that which would be transmitted by an isotropic antenna (transmitting the same power in all directions) placed in the same location. It is usually expressed in *decibels* (dB).

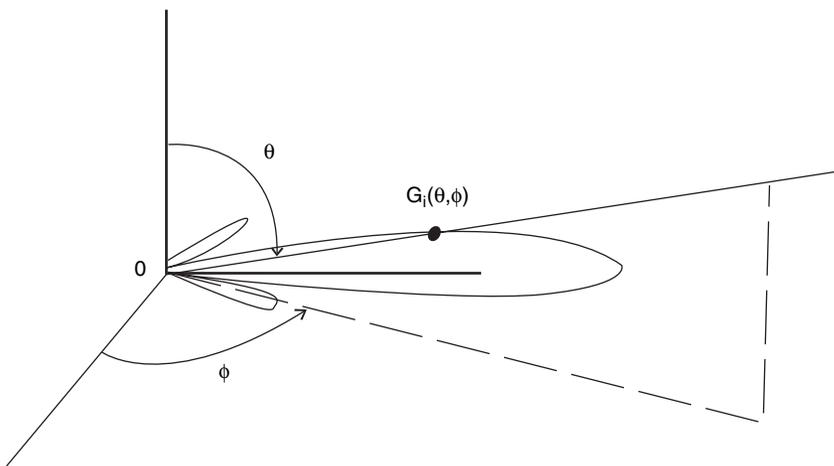


FIGURE 1.5 Radiation diagram of an antenna.

### 1.4.2 Influence of the Waveform

Parameters of microwave exposure are an important consideration in the production of biological effects. One key word is *dosimetry*, which takes into account the level of exposure as well as its duration. The simplest expression is the product of the level and the duration. Different durations of acute exposure lead to different biological effects and, consequently, different long-term effects occur after repeated exposure. The waveform of the radiation is also important. Differential effects have indeed been observed after exposure to pulsed-wave with respect to continuous-wave (CW) microwaves. In practice, biological effects have been observed under a variety of exposure types: CW, sinusoidal amplitude-modulated wave (AMW), pulsed wave (PW), and pulsed modulated wave (PMW) [13].

Hence, there is a difficulty in evaluating the exposure. Thermal effects are of course related to power, so that comparison of biological effects under different types of exposure should be done at constant power. What is constant power, however? Normally it should be either the CW power or the average power when the excitation is pulse modulated. In this case, however, and especially when the duty cycle is short, as in radar-type waves, the peak power may be much larger than the average power, and possible nonlinearity may induce other effects. The difficulty should even be greater for microthermal or non-thermal effects, immediately influenced by fields and not by power. This is a controversial question which shall be considered in Section 3.8. We should not consider power, however, as the only parameter able to induce effects. For instance, differential effects have also been observed after exposure to plane-*versus* circular-polarized waves.

### 1.4.3 Blackbody Radiation

The concept of a *blackbody* radiator is of fundamental importance for the understanding of the emission of real materials, including biological tissues, because its emission spectrum represents a reference relative to which the radiant emittance of a material can be expressed. In general, a fraction of the radiation incident upon a body is absorbed and the remainder is reflected. A blackbody is defined as an idealized, perfectly opaque material that absorbs all the incident radiation at all frequencies, with no reflection: It is a *perfect absorber*. The quantum-mechanical model of a blackbody can be described as consisting of such a large number of quantified energy levels with a correspondingly large number of allowable transitions that any photon, whatever its energy or frequency, is absorbed when incident upon the blackbody. In addition to being a perfect absorber, a blackbody is also a *perfect emitter*, since energy absorbed by a material would increase its temperature if no energy were emitted [14–16]. In other words, a blackbody is lossless. Its behavior is not to be confused, however, with that of a perfect reflector (a mirror), which is lossless too. A photon incident at a given frequency on a perfect reflector is indeed reflected back at the same frequency: The reflected (“emitted”) signal

is perfectly correlated with the incident signal. The perfect reflector may be called a *white body*.

Physically, blackbody radiation is produced by a body that is considered as a closed volume whose walls are in thermal equilibrium at a given temperature. An outside source maintains the whole wall at a constant temperature. If the body is a blackbody, there are no losses and the energy balance is zero. The incident radiation is transformed into thermal agitation, which in turn is transformed into emitted radiation. The emitted signal is perfectly noncorrelated with the incident signal [17]. As a consequence, a blackbody radiates at least as much energy as any other body at the same temperature. At microwave frequencies, good approximations to blackbodies are the highly absorbing materials used in the construction of anechoic chambers. Real materials, such as the earth for instance, usually referred to as *grey bodies*, emit less than blackbodies and do not necessarily absorb all the incident energy. As an example, part of the solar energy incident upon the earth is directly reflected back, so that only part of the incident energy is transformed by a blackbody process and emitted back. As a consequence, the “blackbody temperature” of the earth, that is, the temperature at which a blackbody would emit the same energy as that emitted by the earth, is lower than the physical temperature of the earth; at microwaves, this is about 254 K.

It must be noted that blackbody radiation is usually at a very low level with respect to other radiation sources, in particular power transmitted by communications systems and, especially, by radars. A few examples are worth mentioning. The sun’s total power output is approximately  $3.9 \times 10^{26}$  W, little of which appears at wavelengths below 0.3 nm or above  $3 \mu\text{m}$ . The solar energy per minute falling at right angles on an area of  $1 \text{ cm}^2$  at a solar distance of 1 astronomical distance (equal to the mean earth’s solar distance) is called the *solar constant* and at a point just outside the earth’s atmosphere has a value of  $0.14 \text{ W cm}^{-2} = 1.4 \text{ kW m}^{-2}$ . The planet earth reradiates a part of the received solar energy, equal to the difference between the energy received and the earth’s *albedo* (relative energy absorbed is 0.34 for the earth), equal to  $0.023 \text{ W cm}^{-2} = 230 \text{ W m}^{-2}$ . Cosmic noise extends from about 20 MHz to about 4 GHz. Man-made noise is an unwelcome by-product of electrical machinery and equipment operation and exists from frequencies of about 1 MHz to about 1 GHz. The peak field intensity in industrial areas exceeds the value of cosmic noise by several orders of magnitude, which draws attention to the need for judicious ground station site selection. Blackbody radiation is evaluated later in this section.

Blackbodies will be reevaluated in Chapter 2, devoted to an introduction to biological effects, more precisely hazards due to RF and microwave fields. In that chapter, energy considerations will be compared with entropy considerations while evaluating the possibility of isothermal biological effects.

**Planck’s Radiation Law** Planck established the mathematical expression of the energy emitted by a blackbody in 1901, in his blackbody radiation law, generally termed Planck’s law. It yields the spectral energy density per unit

volume, that is, the energy density per unit volume and unit bandwidth. It can easily be calculated by multiplying the energy per photon by the number of photons in a given mode and by the number of modes in a given volume, which is obtained when using semiclassical statistical mechanics [17]. Planck's law may be expressed in terms of the spectral energy per unit volume. It is often expressed, however, in terms of the *blackbody spectral intensity* per unit solid angle  $I(f; \theta, \phi)$ , also called *blackbody spectral brightness*  $B_f(f; \theta, \phi)$ , where  $f$  is frequency and  $(\theta, \phi)$  characterizes the direction in which the radiation is emitted, in watts per steradian per square meter and per hertz:

$$I(f; \theta, \phi) = B_f(f; \theta, \phi) = \left( \frac{2}{c^2} \right) \frac{hf^3}{\exp(hf/kT) - 1} \quad \text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1} \quad (1.33)$$

where  $f$  is the frequency in hertz;  $T$  the absolute temperature in kelvin;  $h$  Planck's constant =  $6.63 \times 10^{-34}$  J;  $k$  Boltzmann's constant =  $1.380 \times 10^{-23}$  J K<sup>-1</sup>; and  $c$  the velocity of light in a vacuum =  $3 \times 10^8$  m s<sup>-1</sup>. The only two variables in (1.33) are  $f$  and  $T$ . It should be noted that  $hf$  has the dimensions of energy, as well as  $kT$ ; hence the ratio  $hf/kT$  is dimensionless. Figure 1.6 shows a family of curves of  $B_f$  plotted as a function of frequency with temperature as a parameter. It is observed that (1) as the temperature is increased, the overall level of the spectral brightness curve increases, and (2) the frequency at which the spectral brightness is maximum increases with temperature.

**Rayleigh–Jeans Radiation Law** At RFs and microwaves, the product  $hf$  may be very small with respect to  $kT$  so that the denominator of the second factor on the right side of (1.33) can be written as

$$e^{hf/kT} \cong 1 + \frac{hf}{kT} - 1 \cong \frac{hf}{kT} \quad \text{when } hf \ll kT \quad (1.34)$$

which inserted in (1.33) reduces Planck's law to

$$B_f = 2 \left( \frac{f}{c} \right)^2 kT = \frac{2kT}{\lambda^2} \quad \text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1} \quad \text{when } hf \ll kT \quad (1.35)$$

This is the Rayleigh–Jeans radiation law. It is a very useful low-frequency approximation of Planck's law in the radio spectrum. At millimeter waves or at very low absolute temperature, however, one needs to be careful because the condition  $hf \ll kT$  may not always be satisfied. It is worth noting that the ratio  $hf/kT$  is approximately equal to

$$\frac{hf}{kT} \cong \frac{(1/20)f \text{ (GHz)}}{T \text{ (K)}} \quad (1.36)$$

Using the Rayleigh–Jeans approximation yields a definition of the *brightness temperature* of a transmitter. It is the temperature of a blackbody that emits the same brightness (1.35):

$$T_B(f; \theta, \phi) = \frac{\lambda^2}{2k} B_f(f; \theta, \phi) \quad \text{K} \quad \text{when } hf \ll kT \quad (1.37)$$

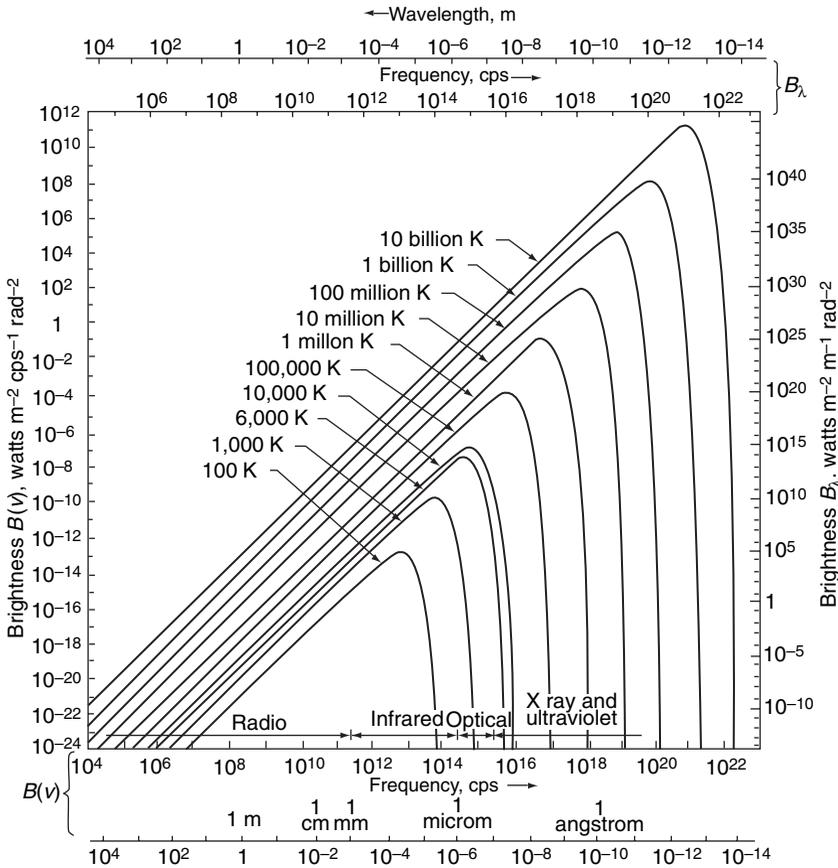


FIGURE 1.6 Planck radiation law curves (from [14]).

According to the Rayleigh–Jeans law, the brightness varies inversely as the square of the wavelength. On a log-log graph this relation appears as a straight line of negative slope. It coincides with Planck’s law for the same temperature at long wavelengths. At short wavelengths, however, the brightness predicted by the Rayleigh–Jeans law would increase without limit, whereas the actual brightness reaches a peak and then decreases with decreasing wavelength, as predicted by Planck’s law.

In fact, the Rayleigh–Jeans law was derived directly by Jeans from classical physics, before Planck introduced his quantum theory [14]. It should be observed that the Rayleigh–Jeans approximation does not contain Planck’s constant  $h$ . It can be shown that this is because Planck’s law is based on the postulate that the radiator possesses only a discrete set of possible energy values or levels, whereas the Rayleigh–Jeans approximation is based on clas-

sical physics, assuming that the radiator may possess all energy levels and that transitions are by a continuous process [17].

The classical *Johnson–Nyquist* expression for the noise emitted by a resistor can easily be derived from the Rayleigh–Jeans approximation. A circuit is formed by connecting a resistor at both ends of a lossless transmission line. Both resistors are equal to the characteristic resistance of the line; hence the line is matched and there are no reflections. The matching resistors and the line are all at the same temperature. Hence, the circuit is a one-dimensional blackbody in which the energy emitted by the resistor at the left propagates on the line and is absorbed by the resistor at the right end, while the energy emitted by the resistor at the right and propagating on the line is absorbed by the resistor at the left. As for establishing the three-dimensional Planck’s law, one multiplies the energy per photon by the number of photons in a given mode and by the number of modes in the given length. The expression is reduced assuming that the Rayleigh–Jeans approximation is valid ( $hf \ll kT$ ). Dividing by 2 to obtain the spectral power density for one resistor only yields the classical expression for the noise energy emitted by a resistor:

$$S(f) = kT \quad \text{W Hz}^{-1} \quad (1.38)$$

It is interesting to observe that this expression is independent of the value of the resistor: Two resistors, with values  $1\ \Omega$  and  $1\ \text{M}\Omega$ , respectively, produce the same energy. It is important to note that expression (1.38) is valid only under the Rayleigh–Jeans approximation. The power emitted in a given bandwidth  $B$  is of course

$$P = kTB \quad \text{W} \quad (1.39)$$

From this expression one can define the noise temperature of a signal: It is the temperature at which a pure resistor should be maintained to produce the same power spectrum:

$$T(f) = \frac{S(f)}{k} \quad \text{K} \quad (1.40)$$

In particular, this is the adequate definition for the *antenna temperature*, which is the absolute temperature at which a pure resistor must be maintained to produce the same power density as that measured at the output of the antenna. In other words, if a pure resistor maintained at the *antenna temperature* replaces the antenna, the receiver does not observe a difference. One then uses expression (1.39) for calculating the noise power emitted by a pure resistor, considered as the source of a generator with the resistor as the series resistance. A resistor of the same value to match the source terminates this source. It is well known that the generator then delivers the maximum possible power. Calculating this power yields

$$\langle e^2(t) \rangle = 4RkTB \quad \text{when } hf \ll kT \quad (1.41)$$

where the brackets mean the average value. This is the well-known expression for the Johnson–Nyquist noise emitted by a resistor. This expression depends upon the value of the resistor and its range of application is limited because of the limitations of the Rayleigh–Jeans approximation.

It is interesting to calculate the blackbody radiation and compare it with other radiation sources evaluated earlier in this section. Equation (1.40) can be used in most cases. It shows that the noise power of a blackbody source at a physical absolute temperature of 300K and measured in a bandwidth of 10MHz is  $41.4 \times 10^{-15}$ W. If the source is at 3000K and the receiver bandwidth is 100MHz, the power is  $41.4 \times 10^{-13}$ W. It appears that the blackbody radiation is at a power level much smaller than most of the physical and industrial sources.

**Stefan–Boltzmann Law** Planck’s law predicts the brightness curves presented in Figure 1.6 as a function of frequency and temperature. Integrating the Planck radiation law over all frequencies, which is summing the area of the Planck radiation law curve for that temperature, yields the *total brightness*  $B_t$  for a blackbody radiator:

$$B_t = \left( \frac{2h}{c^2} \right) \int_0^\infty \frac{f^3}{e^{hf/kT} - 1} df \quad \text{W sr}^{-1} \text{ m}^{-2} \quad (1.42)$$

The integration yields the Stefan–Boltzmann relation [15]

$$B_t = \sigma T^4 \quad \text{W sr}^{-1} \text{ m}^{-2} \quad (1.43)$$

where  $B_t$  is the total brightness,  $\sigma$  a constant equal to  $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ , and  $T$  the absolute blackbody temperature in kelvin. It must be observed that the *total* brightness of a blackbody increases as the fourth power of its temperature. As an example, the total brightness of a blackbody at 1000K should be 16 times that of at 500K. This temperature dependence is valid only for the total brightness and not for the brightness (1.33) or (1.35).

**Wien Displacement Law** Planck’s law represented in Figure 1.6 shows that the peak brightness shifts to higher frequencies with an increase in temperature. Maximizing Eqn. (1.33) by differentiating with respect to frequency  $f$ , that is, in terms of unit frequency, and setting the result equal to zero yield a quantitative expression for this displacement at a specific frequency  $f_p$ . Noting that the peak occurs at high values of the ratio  $hf_p/kT$  and simplifying accordingly yield the expression of the wavelength  $\lambda_p$  at which  $B$  is a maximum [15]:

$$\lambda_p T = \frac{hc}{3k} = 0.0048 \quad \text{m K} \quad (1.44)$$

This relation, in which the product of wavelength and temperature is a constant, is called the *Wien displacement law*: The wavelength of the maximum of

peak brightness varies inversely with the temperature. Avoiding the simplification related to the high value of the ratio  $hf_p/kT$  yields the exact value of 0.0051 mK for the constant.

Instead of differentiating with respect to frequency, one can also differentiate with respect to wavelength, that is, in terms of unit wavelength. The brightness is then expressed in terms of unit wavelength and the wavelength for the peak brightness is not the same as when brightness is expressed in terms of unit bandwidth. The quantitative relation for the  $\lambda_p T$  product when brightness is expressed in terms of unit wavelength is obtained by maximizing (1.42) and simplifying, which yield the value 0.0029 mK [15].

The frequency  $f_p$  at which the maximum radiation occurs increases as the temperature increases. Further calculations [14] yield the following expression for this frequency:

$$f_p = 5.87 \times 10^{10} T \quad \text{Hz} \quad (T \text{ in kelvin}) \quad (1.45)$$

and for the maximum spectral brightness  $B_f$  with respect for  $f$ :

$$B_f f_p = a T^3 \quad \text{W sr}^{-1} \text{ m}^{-2} \text{ Hz}^{-1} \quad (1.46)$$

where  $a = 1.37 \times 10^{-19} \text{ W sr}^{-1} \text{ m}^{-2} \text{ Hz}^{-1} \text{ K}^{-3}$ . Expressions (1.45) and (1.46) are valid for the spectral brightness per unit frequency. They are not valid for the spectral brightness per unit wavelength, for which similar expressions can be obtained.

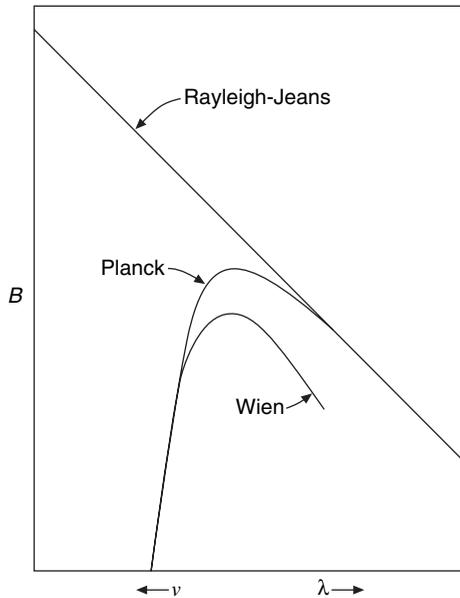
**Wien Radiation Law** The Rayleigh–Jeans radiation law (1.35) is a low-frequency approximation of Planck’s radiation law (1.33), valid when one has  $hf \ll kT$ . At high frequencies, when one has  $hf \gg kT$ , the quantity unity in the denominator of the second factor on the right side of (1.33) can be neglected, in comparison with  $e^{hf/kT}$ , so that Planck’s law reduces to

$$B_f = \left( \frac{2hf^3}{c^2} \right) e^{hf/kT} \quad \text{W sr}^{-1} \text{ m}^{-2} \text{ Hz}^{-1} \quad (1.47)$$

called the *Wien radiation law*. The Wien law coincides with Planck’s law for the same temperature at frequencies considerably higher than the frequency of maximum radiation. A comparison of Planck’s law with its high- (Wien) and low-frequency (Rayleigh–Jeans) approximations is provided in Figure 1.7 [14]. The coincidence of both approximations with the corresponding part of Planck’s law is obvious.

## 1.5 PENETRATION IN BIOLOGICAL TISSUES AND SKIN EFFECT

When a conductive material is exposed to an EM field, it is submitted to a current density caused by moving charges. In solids, the current is limited by the collision of electrons moving in a network of positive ions. Good conduc-



**FIGURE 1.7** Comparison of Planck's law with its high- (Wien) and low-frequency (Rayleigh-Jeans) approximations at 300K (from [15], courtesy of J. D. Krause, Jr.).

tors such as gold, silver, and copper are those in which the density of free charges is negligible, the conduction current is proportional to the electric field through the conductivity, and the displacement current is negligible with respect to the conduction current. The propagation of an EM wave inside such a material is governed by the diffusion equation, to which Maxwell's equations reduce in this case. Biological materials are not good conductors. They do conduct a current, however, because the losses can be significant: They cannot be considered as lossless.

Solving the diffusion equation, which is valid mainly for good conductors, where the conduction current is large with respect to the displacement current, shows that the amplitude of the fields decays exponentially inside of the material, with the decay parameter

$$\delta = \frac{1}{(\omega\mu\sigma/2)^{1/2}} \quad \text{m} \quad (1.48a)$$

The parameter  $\delta$  is called the *skin depth*. It is equal to the distance within the material at which the fields reduce to  $1/2.7$  (approximately 37%) of the value they have at the interface. One main remark is that the skin depth decreases when the frequency increases, being inversely proportional to the square root of frequency. It also decreases when the conductivity increases: The skin depth

is smaller in a good conductor than in another material. Furthermore, it can be shown that the fields have a phase lag equal to  $z/\delta$  at depth  $z$ .

For most biological materials the displacement current is of the order of the conduction current over a wide frequency range. When this is the case, a more general expression should then be used instead of (1.48a) [16]:

$$\delta = \left( \frac{1}{\omega} \right) \left\{ \left( \frac{\mu \varepsilon}{2} \right) \left[ (1 + p^2)^{1/2} - 1 \right] \right\}^{1/2} \quad (1.48b)$$

where  $p = \sigma/\omega\varepsilon$  is the ratio of the amplitudes of the conduction current to the displacement current. It is easily verified that Eqn. (1.48b) reduces to Eqn. (1.48a) when  $p$  is large.

The following important observations can be deduced from Eqn. (1.48a):

1. The fields exist in every point of the material.
2. The field amplitude decays exponentially when the depth increases.
3. The skin depth decreases when the frequency, the permeability, and the conductivity of the material increase. For instance, the skin depth of copper is about 10 mm at 50 Hz, 3 mm at 1 kHz, and  $3 \mu\text{m}$  at 1 GHz. It is equal to 1.5 cm at 900 MHz and of the order of 1 mm at 100 GHz in living tissues.

These results are strictly valid for solids limited by plane boundaries. They are applicable to materials limited by curved boundaries when the curvature radius is more than five times larger than the skin depth. In the other cases, a correction has to be applied.

The phenomenon just described is the *skin effect*: Fields, currents, and charges concentrate near the surface of a conducting material. This is a shielding effect: At a depth of  $3\delta$ , the field amplitude is only 5% of its amplitude at the interface, and the corresponding power is only 0.25%; at a depth of  $5\delta$ , the field amplitude reduces to 1% and the corresponding power to  $10^{-4}$ , which is an isolation of 40 dB. This shows that, at extremely low frequency, for instance at 50 Hz, it is illusory to try to shield a transformer with a copper plate: A plate 5 cm thick would be necessary to reduce the field to 1%! This is the reason why materials which are simultaneously magnetic and conducting, such as mumetal, are used for low-frequency shielding. In practice, the skin effect becomes significant for humans and larger vertebrates at frequencies above 10 MHz.

Shielding is much easier to achieve at higher frequencies. The skin effect implies that, when using microwaves for a medical application, the higher the frequency, the smaller the penetration, which may lower the efficiency of the application. Hence, the choice of frequency is important. It also implies that if a human being, for instance, is submitted to a microwave field, the internal organs are more protected at higher than at lower frequencies. As an example, the skin depth is three times smaller at 900 MHz, a mobile telephony fre-

quency, than at 100 MHz, an FM radio frequency, which means that the fields are three times more concentrated near the surface of the body at 900 MHz than at 100 MHz. It also means that internal organs of the body are submitted to higher fields at lower than at higher frequency.

The skin effect is well known in engineering. It is also characterized by the *intrinsic* (or *internal* or *metal*) *impedance*, obtained by dividing the electric field at the surface by the current per unit width flowing into the material:

$$Z_m = \frac{E_0}{I_x} = (1 + j)\sigma\delta = \sqrt{\frac{j\omega}{\sigma}} \quad \Omega \quad (1.49)$$

As an example, the intrinsic impedance of copper at 10 GHz is  $0.026(1 + j) \Omega$ . It has equal real and imaginary terms. The real part  $R_m$  is sometimes called the *surface resistance*, measured in *ohms per square*.

The impedance is useful for calculating the (complex) power dissipation in the material. A property of the exponential curve is that its integration yields a result equal to the initial value times the skin depth. In other words, the intrinsic resistance of the material in which the fields decay exponentially would be the same if the current was uniformly distributed over depth  $\delta$ . Hence, the total power dissipated in the material under an exponentially decaying field is equal to the power dissipated in depth  $\delta$  under a field constant and equal to the value at the surface.

Table 1.2 summarizes some skin depth values for human tissues at some frequencies. The EM properties of the tissues as well as their variation as a function of frequency have been taken into account.

Figure 1.8 shows the variation of the power absorbed inside a human body as a function of the penetration depth at several microwave frequencies: We are less and less transparent to nonionizing EM radiation when the frequency increases. In the optical range, skin depth is extremely small: We are not transparent anymore. Variation of the dielectric constant as a function of frequency was taken into account in this figure.

There is a tendency to believe that RFs and microwaves exert more significant biological effects at low and extremely low frequencies. This is not necessarily true: The dielectric constant of living materials is about 10,000 times larger at ELF than at microwaves. The dielectric constant is important because

**TABLE 1.2 Typical Skin Depths in Human Tissue**

Parameter	Radio FM	TV Transmitter	Telephony Mobile	Telephony Mobile
Frequency (MHz)	100	450	900	1800
Skin depth (cm)	3	1.5	1	0.7
Depth at which power reduces to 1% (cm)	9	4.5	3	2

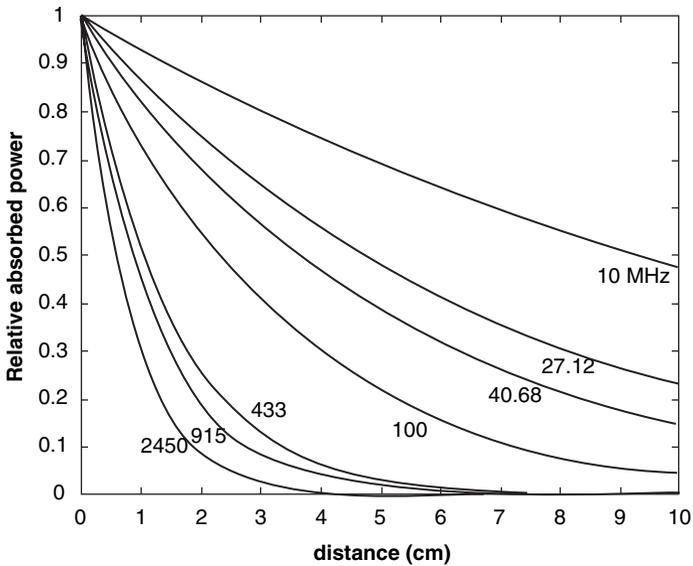


FIGURE 1.8 Power absorbed in muscles as a function of the skin depth at various frequencies.

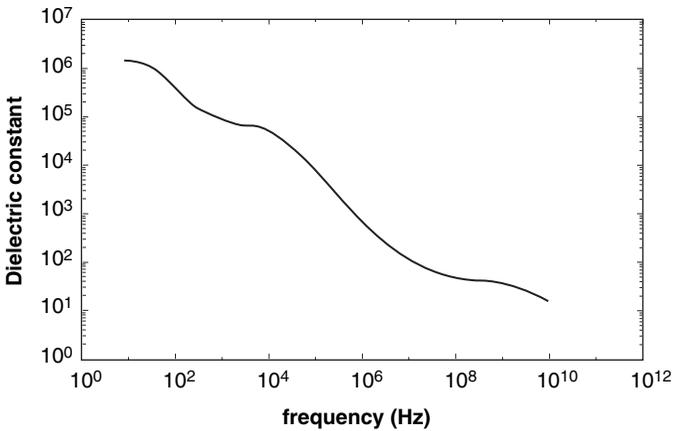


FIGURE 1.9 Dielectric constant of living material as a function of frequency.

it is the link between the source field and the electric flux density (also called the displacement field). A dielectric constant 10,000 larger implies the possibility of an electric flux density of a given value with a source field 10,000 times smaller. Figure 1.9 shows the dielectric constant of living material (muscle) as a function of frequency [18]. There is a level of about 1,000,000 at ELF up to 100 Hz, then a second level of about 100,000 from 100 Hz to 10 kHz, and, after

some slow decrease, a third level of about 70–80 from 100 MHz to some gigahertz. This last value is that of the dielectric constant of water at microwaves. One of the main constituents of human tissues is water. Hence, we have about the same microwave properties as water.

However, as has been said in Section 1.1, microwaves are in the frequency range in which the wavelength is of the order of the size of objects of common use—meter, decimeter, centimeter, and millimeter—depending of course on the material in which it is measured. One may hence wonder whether such wavelengths can excite resonance in biological tissues and systems. We shall come back with this question in Chapters 2 and 3.

## 1.6 RELAXATION, RESONANCE, AND DISPLAY

A good knowledge of the complex permittivity of biological media is necessary for evaluating biological effects as well as in medical applications. Hyperthermia is only one example of such an application (treated in great detail in Chapter 4). A number of measured data are available for characterizing biological media. It should be mentioned, however, that there are not many measured data for biological and organic liquids at frequencies above 20 GHz.

### 1.6.1 Relaxation in Dielectrics

The dielectric constant is the real part of the complex relative permittivity. It is of primary importance when characterizing dielectrics. It should not be forgotten, however, that the permittivity is complex in the frequency domain and that the dielectric constant gives only partial information.

Up to about 1 GHz, materials respond to *relaxation* phenomena, already illustrated in Section 1.3.1. When a rarefied *nonpolar gas* in which the molecules have no electric dipoles at rest is submitted to an external electric field, an electric dipole is induced. When a rarefied *polar gas* which has an electric dipole at rest is submitted to an external electric field, the dipole orientation is modified: It essentially rotates. As has been said earlier, when the density increases, classical physics almost completely fails when trying to establish quantitative models. It can, however, yield some insight on the phenomena involved with the dielectric character of materials.

The alignment of the molecule dipolar moment because of an applied field, called *dipolar polarization*, is a rather slow phenomenon. It is correctly described by a first-order equation, called after Debye [4, 5]: The dipolar polarization reaches its saturation value only after some time, called the *relaxation time*  $\tau$  (Fig. 1.1).

This looks simple. The process is rather complicated, however. The task of dielectric theory is difficult, not so much because permanent dipoles cannot always be identified but mainly because they mutually influence one another: A dipole not only is subject to the influence of a field but also has a field of

its own. The mutuality of the influence of dipoles, permanent or otherwise, on one another makes the response of the assembly a cooperative phenomenon, depending on the size and shape of the assembly [5].

The relaxation process consists in the approach to equilibrium of a system which is initially not in thermodynamic equilibrium. Such a process is irreversible and not covered by equilibrium thermodynamics. Most relaxation processes fulfill the conditions for which irreversible thermodynamics is applicable. Relaxation occurs when the free energy stored in the system is degraded into heat, in other words, if entropy is created irreversibly. The irreversibility is related to the fact that the free energy of the field is used to increase the total amount of heat stored in the dielectric plus the heat reservoir surrounding it. The thermodynamic treatment illuminates the significance of some of the concepts of relaxation and shows, in particular, that the Debye equation is the most plausible description of a relaxation process in a first approximation. A more thorough understanding requires further reading [5].

What has just been described corresponds to dielectrics with a single relaxation time. In fact, three classes of dielectric characteristics may be distinguished:

1. *Dielectrics with Single Relaxation Time.* These are rare. For instance, they are found in certain dilute solutions of large polar molecules in nonpolar solvents. Some other liquids and solids obey the Debye equation for complicated reasons.
2. *Dielectrics with Approximate Debye Behavior.* The peaks of  $\epsilon''$  are wider and have a somewhat different shape. This is a large class which contains many simple compounds and solutions of dipolar compounds in nonpolar solvents.
3. *Dielectrics for Which Debye Peaks Are Unrecognizable.* This class contains many practical insulating materials as well as some pure compounds.

For materials with many relaxation times, the polarizable elements are divided into groups of given relaxation times; then a principle of superposition is used to obtain an analytical expression for the complex permittivity.

Few substances have one single relaxation time. For some materials, such as water, permittivity is closely fit by a first-order Debye equation, while for others, such as biological media, where several relaxation times are involved, higher order terms are necessary [19].

### 1.6.2 Resonance Absorption

Relaxation refers to that part of polarization that is due to the ordering of permanent dipoles. However, matter can take energy from a field even in the absence of permanent dipoles if the field perturbs oscillations of one kind or

another. The polarization caused by this mechanism is called *electronic polarization* or *optical polarization*.

Resonance can be observed in gases at frequencies below 100 GHz, although, in condensed matter, such an effect can be observed optically in the IR region of the spectrum. Besides, in gases, optical polarization may be described as a property of individual molecules, while conditions in solids and liquids are less easy to visualize.

An individual atom or molecule, whether it has a permanent dipole moment or not, consists of negative and positive charges. In terms of classical physics, it may be considered as a harmonic oscillator: Overcoming a restoring force can alter the distance between the centers of gravity of the positive and negative charges. The oscillator takes energy from an electric field, at a resonant frequency, determined by the restoring force; the energy uptake in the absence of damping tends to infinity.

The interaction of molecules and fields is best described by quantum mechanics [5]. Individual molecules have discrete energy states and they absorb energy from an alternating field of frequency  $f$ , so that

$$hf = \Delta W \quad \text{J} \quad (1.50)$$

where  $\Delta W$  is the energy difference between two quantum levels and  $h$  is Planck's constant. The molecule can take up or radiate energy only at the appropriate frequencies and in the appropriate amounts determined by Eqn. (1.50). Each individual molecule is characterized by an optical spectrum with a number of discrete frequencies. The energy differences are extremely small and the smallness of  $\Delta W$  may be appreciated when it is compared with  $kT$ : For  $f = 10$  GHz,  $\Delta W$  is only  $(6 \times 10^{-3})kT$  at room temperature while the energy per degree of freedom is  $1/2 kT$  at equilibrium. Detailed investigation of the spectra for gases at low pressures is provided by microwave spectroscopy.

When molecules exist within a gas or other assembly, they collide or otherwise interact with neighbors. These interactions result in thermal equilibrium, which implies an equilibrium distribution of energies among the constituent members of the assembly. It also implies that a given molecule retains its energy only for a certain average time  $\tau_i$ . In a gas, this lifetime is shorter when the pressure is higher.

When measuring the value of an energy level in an assembly of molecules by means of some external probe, for instance microwaves, the result is unprecise according to the Heisenberg uncertainty principle. If this energy level has a lifetime  $\tau_i$ , then it can only be determined with an uncertainty  $\delta W$  given by

$$\delta W \cong \frac{h}{\tau_i} \quad (1.51)$$

When  $\tau_i$  is short, the uncertainty of the energy determination may be large compared with the magnitude of the energy difference to be determined. In

this case, the magnitude of  $\Delta W$  cannot be determined, and spectroscopy is impossible.

Neighboring oscillators interchange energy by collisions. Lorentz deduced a theory of line broadening which is a good approximation for the case that neither the resonance frequency nor the field frequency is very small and that line broadening is not too great, which covers most typical spectra [20]. His results can be closely approximated by introducing a damping term in the equation of the oscillator which yields the very simple solution of a second-order differential equation with a damping term, the simplest formal description of resonance absorption. The behavior of the complex dielectric constant as a function of frequency is quite different for resonance and relaxation, as may be seen from Figures 1.1 and 1.2.

The absorption spectra of crystalline solids are at short wavelengths due to electronic transitions within atoms or molecules and at longer wavelengths due to vibrations of the crystal as a whole [5]. The lattice vibrations are either optical or acoustic. Together they constitute the thermal vibrations whereby the atoms and ions, for example, within the crystal achieve thermal equilibrium. The acoustic vibrations do not involve a change of the polarization and do not interact with EM fields. The optical vibrations correspond to oscillatory displacements of charges.

The most important optical vibration in a simple ionic crystal such as KCl may be visualized as a movement of K and Cl ions in opposition to each other, as in the case of a dipole whose length is oscillating. The frequency of this vibration in the alkali halides is of the order of 10 THz, in the IR spectrum. Investigating the broadening of the IR spectra of crystalline solids is a difficult subject, although first-order approximations can be obtained rather easily.

The transition from resonant to nonresonant absorption, that is, relaxation, has been studied in gases by increasing damping, hence line broadening. It has also been approached for solids. When the resonant frequency decreases, however, optical measurements become increasingly difficult and very complex techniques have to be used. For a wavelength of about 1 mm, which corresponds to 300 GHz in a vacuum, the optical techniques of the far-IR overlap with microwave techniques.

### 1.6.3 Cole–Cole Display

The data provided by dielectric measurements can be presented in different ways. One classical representation is in plotting the real and imaginary parts of the permittivity as functions of frequency, most frequently as functions of  $\log_{10} \omega$ . The disadvantage, however, is that these two plots are then presented independently of each other while their frequency behaviors are linked through the general theoretical considerations. These were developed in Section 1.3.1 where it was shown that each part can be calculated from the variation of the other part over the whole frequency range, as indicated by the Kramer and Kronig equation (1.12) [6].

One very early representation is the plot of  $\epsilon''(\omega)$  for a certain frequency against  $\epsilon'(\omega)$  at the same frequency, which is the representation of the dielectric constant in its complex plane. This display is often called the *Cole–Cole plot* (or *complex-locus diagram* or *Argand diagram*) [5]. Using expressions (1.9), it can easily be shown that Debye’s equation between the real and imaginary parts of the dielectric constant is the equation of a circle in the complex plane  $\epsilon(\omega)$ :

$$[\epsilon'(\omega) - \frac{1}{2}(\epsilon_s + \epsilon_\infty)]^2 + [\epsilon''(\omega)]^2 = \frac{1}{4}(\epsilon_s - \epsilon_\infty)^2 \tag{1.52}$$

where  $\epsilon_s$  and  $\epsilon_\infty$  are the values of the real part of the relative permittivity at frequencies zero and infinity, respectively. The Cole-Cole plot therefore provides an elegant method of finding out whether a system has a single relaxation time. Figure 1.10 shows the plot of a dielectric with a single relaxation time. A given point on the semicircle corresponds to a given frequency. Frequency is zero on the right end of the diameter and infinity on the left, increasing anticlockwise. The summit corresponds to  $\omega\tau = 1$ , where  $\tau$  is the relaxation time in Eqn. (1.7). The plot has the disadvantage that  $\omega$  does not appear explic-

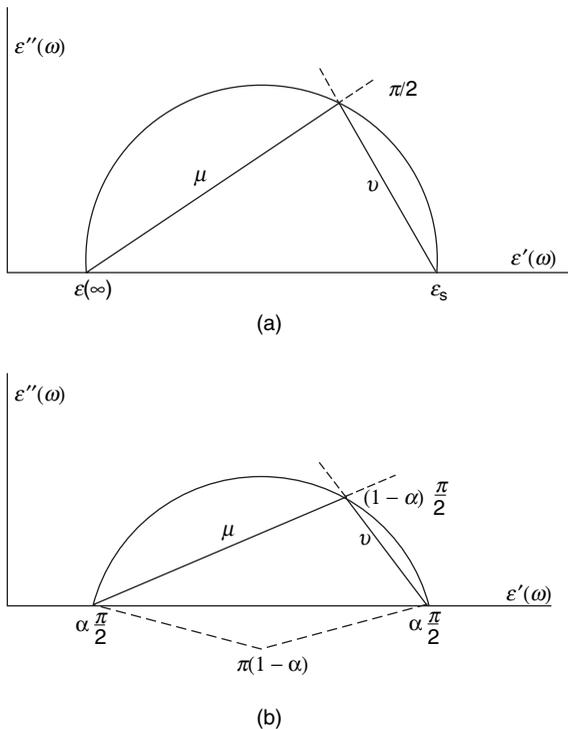


FIGURE 1.10 Cole–Cole plot for a Debye dielectric.

itly: Any material with a single relaxation time, characterized by  $\epsilon_s$  and  $\epsilon_\infty$ , gives the same plot. Hence, in reporting results, it is essential to supply the magnitude of  $\tau$  in addition to the plot. The power at which  $\omega\tau$  appears in Eqn. (1.7) is not always unity. When it is smaller than unity, the plot represents only an upper portion of a semicircle.

When the steady conductivity contributes significantly to  $\epsilon''(\omega)$ , the representation of dielectric data is affected in the Cole–Cole plot [5]. The presence of several relaxation times also complicates the representation. In view of this, Fröhlich has introduced a distribution function in which the relaxation time depends on the (absolute) temperature, and he assumes that the polarizable units are evenly distributed in terms of activation energy [21]. There is no reason why activation energies should be evenly distributed over a small range of values while higher or lower values should be absent. Fröhlich's distribution can be generalized by assuming that most activation energies have a median value while deviant values are increasingly unlikely, yielding some sort of a Gaussian distribution of activation energies [5].

However, Cole and Cole designed a function that is not very different in practice from a Gaussian distribution. It is a useful representation for many experimental results, and this seems reasonable in view of its similarity to a Gaussian distribution. Several other distribution functions were designed later.

## 1.7 DIELECTRIC MEASUREMENTS

Accurate knowledge of the complex relative permittivity of biological tissues is necessary to evaluate biological effects as well as the efficiency of medical applications. There are a variety of measurement methods, based however on a rather limited number of principles.

### 1.7.1 RF Measurements

The complex permittivity—dielectric constant and losses—of a material is almost always measured by inserting a specimen into a capacitor, waveguide, or cavity which forms part of an electrical circuit. The circuit is subject to an alternating voltage or wave or to step functions. Above 1 MHz, however, which covers the RF/microwave range, the source delivers most generally a CW. In all cases, errors may arise if effects due to some part of the external circuit are ascribed to the specimen. It is extremely important to point out the limits of experimental accuracy. Furthermore, some sources of error might escape the experimenter when using ready-made equipment.

For frequencies higher than a few megahertz, representation of a circuit by discrete circuit elements and by wires free of inductance and capacitance becomes gradually unrealistic. For higher frequencies, circuits are considered as distributed impedances, and the specimen is incorporated accordingly.

For frequencies up to 1 GHz resonant circuits may be used, the specimen being combined with a known inductance in the form of a  $Q$ -meter using essentially a series combination of  $C$ ,  $L$ , and  $R$  [5]. The energy absorption exhibits a resonant peak and the width of this peak  $2\Delta\omega$  is measured, where  $\Delta\omega$  is the difference between the frequency where the energy loss is a maximum and the frequency where it is reduced to half the maximum. For a simple series circuit the  $Q$  value is given by

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \quad (1.53)$$

Insofar as the capacitor is the only lossy component,  $Q$  is inversely proportional to  $\tan \delta$  of the dielectric at  $\omega_0$ .  $Q$ -meters which use leads from instrument to specimen may be inexpensive and versatile, but they are generally not suitable for precision measurements of loss angle. Leads can be avoided, for instance, by using a capacitance whose value can be modified by micrometer screws. This, however, makes the method difficult to use over more than a narrow range of temperatures.

### 1.7.2 Microwave Measurements

At frequencies above 300 MHz, the shape and size of the measuring assembly may become important in relation to the wavelength. Because of this, different specimens have to be used for not so very different frequencies. Measurements in this microwave region are discussed in a number of textbooks, essentially with nonautomated equipment however [22–24].

In the early days of microwave measurements, accurate measurements were made with the use of a slotted line. In practice, this reduces to measuring the amplitude and phase of an impedance at a given frequency as a function of the position on the line. The slotted line is still the most accurate measurement device. It is not in use anymore, however, because it is limited to one single frequency at a time and making measurements over a range of frequencies necessitates a number of tedious adjustments. Because of transmission line properties, however, the important propagation term is the product of the propagation constant and the position on the line. Hence, operating at constant frequency as a function of the position on the line yields the same type of results as operating at a given point on the line as a function of frequency.

This is what most measuring devices have been able to accomplish since about 1990. They are called *vector analyzers*, and they yield the amplitude and phase of an impedance in a specific point of a line as a function of frequency, for instance from 40 MHz to 40 GHz and higher. The calibration of these instruments is extremely important: It is a key element in doing good measurements. It takes into account all the elements included in the measurement process of the various components outside of the device under test: mismatches, losses, defects, imperfections, and so on. Calibration compensates all these, so that the actual measurements are not degraded. Some devices with

known properties are necessary for calibration purpose, such as a short circuit, an open circuit, and transmission lines with different lengths.

A poor calibration may degrade the measurements without making the experimenter aware of this. Because there are a variety of devices to be measured, there are also a number of calibration procedures. They become more complicated when the number of imperfections to be corrected increases. For instance, the number of terms needed for calibrating the measurement setup depends upon the fact that the physical quantity to be measured is a reflection or a transmission, on a device with small or high losses, reciprocal or not, and so on.

Scattering matrix parameters are most often used for calculating or estimating the errors. They describe a two-port by its reflection and transmission characteristics. When measuring the electric properties of a dielectric, the container in which the specimen is placed is considered as the electrical two-port.

A number of precautions have to be taken, especially at the higher frequencies, above 20 GHz [17]. In particular, connectors can be a source of inaccuracy if not adequately chosen, especially the miniature connectors introduced around 1960. Better connectors are necessary when operating at higher frequencies.

Measuring the complex permittivity of biological tissues fortunately does not offer the same variety as can be found with electronic circuits, passive and active. However, one needs to be careful sometimes: Anisotropy may be present because of the inhomogeneity of the structure. In such a case, calibration has to take the possibility of anisotropy into account.

### 1.7.3 Liquids

Measuring the dielectric properties of liquids raises specific difficulties. In particular, possibly the container must be reusable while being tight. The usual practice for liquids is to measure the reflection coefficient of an open-ended coaxial probe [25]. Two difficulties, however, relate to this practice, especially when applied in the high range of microwave frequencies: The structure radiates and the size of the coaxial probe is very small; hence the mechanical accuracy may not be very good.

A new procedure for measuring the complex permittivity of liquids was presented in 1997 [26]. It is based on the measurement of the scattering parameters of waveguide two-ports and on an original calibration method developed by the authors, called line–line (LL) [27]. The complex permittivity is obtained explicitly. Hence the method avoids the difficulties related to the inversion of implicit expressions through tedious numerical iterations.

A waveguide transmission method was used on a vector analyzer, yielding the four complex scattering parameters: reflection at both ends and transmission in both ways. A waveguide spacer is placed between the two waveguide ends of the vector analyzer. The connection between the waveguides and both ends of the analyzer is made by coax-to-waveguide transitions specific to each

frequency band. To use the LL method for calibration purposes, two waveguide spacers that are identical except for their thickness are necessary. The LL method requires a “short” and a “long” line, respectively. A synthetic film is placed at both ends between waveguides and spacers, containing the liquid in a known volume. Reference planes are placed at both ends in the plane of each film. The calibration method ensures that the transition flange–film–flange has no effect on the result, provided this transition is reproducible when using both spacers. Commercially available simple configurations are adequate. In practice, the film is put on both sides of the waveguide spacer and soldered together. Holes are made in the film, in the place of the holes of the waveguide flanges, to ensure tightness. Then, using a syringe, the liquid is inserted into the spacer. Air bubbles must be avoided. The two waveguides are then screwed together, through the holes of the spacer. This operation has to be done for each of the two spacers and for each waveguide band.

There are several advantages to these methods. First, the test cell is made of commercially available devices. Second, only two operations are needed to extract the required experimental data, and explicit expressions provide the values of the complex permittivity. This is especially helpful for characterizing highly dispersive and lossy substances. Finally, the influence of the air–liquid interface is removed from the procedure, so that mismatch, radiation, and higher order mode generation have no significant influence on the result.

The measured liquids were dioxane, methanol, blood, and axoplasm. Dioxane is a perfect dielectric, with a dielectric constant of about 2.25 and no losses up to 110 GHz. It is extremely useful as a calibration liquid dielectric to check the validity of the measurement setup and, in particular, the quality of matching the various waveguides. Results showed for the first time the complex permittivity of biological and organic liquids at frequencies from 8 to 110 GHz in five waveguide bands. The comparison has been made between the measurements and the first-order Debye equation. For some materials, such as water, permittivity is closely fit by a first-order Debye equation, while for others, such as biological media, where several relaxation times are involved, higher order terms are necessary [19]. Two relaxation times were exhibited by the measurements on blood.

The configuration just described is completely closed, which provides very accurate results. One may need or wish to use open structures. For instance, the complex permittivity of live and dead neurological cell cultures has been measured on a setup made of open microstrip transmission lines, from 20 to 40 GHz [28]. One advantage of this configuration is that it is open. Hence, the biological medium can be submitted to a microwave exposure and measurement of the complex permittivity can be an indicator of the activity of the medium.

#### 1.7.4 Applicators

The applicators usually operate in the near-field region of the antenna, where the biological tissue, or part of the human body, is placed. One needs to be

very careful about how to match an applicator to the part of interest of the human body. Furthermore, the dielectric parameters of the tissue exposed to the antenna are probably not too well known, because they have most certainly been measured in another configuration. It would be very difficult to measure with some accuracy the parameters of a flat living surface, in particular because it would be extremely difficult, if not impossible, to design a calibration method to be used in this configuration. As a consequence, matching an applicator to a biological surface is a very specific subject. The reader is referred to specific further reading, for instance [29].

## 1.8 EXPOSURE

The importance of evaluating correctly the electric parameters of biological tissues has been stressed in this chapter. This is because such knowledge is necessary in evaluating biological effects, in particular in medical applications. This will be considered in detail in the last chapters. One must, however, realize already at this stage that biological effects are to be evaluated with respect to fields that are inside the biological material, and this implies a good knowledge of both the field at the interface with the material and the material properties. In Section 1.7 were exposed the main guidelines for measuring the electric properties of biological materials, together with the precautions to be taken to ensure a good knowledge at the interface.

A totally different situation occurs in real life, when human beings are exposed to a surrounding RF or microwave field due to a variety of sources: AM radio, FM radio, television, mobile telephony, radars, and so on. Similarly, animals prepared for epidemiological studies are submitted to a specific field. In both cases it is necessary to have an accurate knowledge of the field amplitude, for instance, for comparing the human exposure to standards or evaluating possible hazards, the risk of interference in a hospital, the field amplitude at which the animals will be submitted in the epidemiological study, and so on.

Standards for human beings are based on two kinds of limitation:

1. *Basic limitations* that should always be respected.
2. *Reference levels* that could be exceeded if the basic limitations are not exceeded.

The basic limitations are expressed in terms of absorption by the human body, more precisely by the part of the body that is exposed since, because of the skin effect, only the most external layer is absorbing the power. The biological aspects of this will be considered in Chapter 2. It is quite appropriate, however, to note that this power absorption is expressed in *watts per kilo* (W/kg) of absorbing matter. This quantity is called the *specific absorption rate* (SAR). The SAR is of course not measured on a living person.

This is why there are reference levels, which are expressed in measurable physical quantities, namely the fields, electric or magnetic. At RF/microwaves,

the electric field is mostly used as a reference. Standards then specify the value of the electric field, for instance, which should not be exceeded in some place or for some time. This creates a situation that should be well understood: The reference level is a level that the reference, for instance the electric field, should not exceed. This value is measured in the *absence* of the person. The corresponding basic limitation then indicates the power absorbed by a person that should be *present* in this electric field, measured in the absence of the person.

Hence, correctly measuring the electric field or the associated EM power is of prime importance. The basic theory has been exposed in the first sections of this chapter. It should be stressed, however, that in most circumstances the electric field can vary quite significantly from one place to another, not far away from the first: The *variability in space* may be quite important. This may happen, for instance, because of standing waves caused by steady obstacles. If the obstacles are moving, for instance cars in a street, *variability in time* is added. The difficulty of correctly evaluating a living system should not be underestimated.

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## PROBLEMS

- 1.1.** In a nonmagnetic and homogeneous lossy medium, the electric field intensity of a plane wave is  $E = E_0 e^{-\gamma z}$ , where  $\gamma = \alpha + j\beta$ . The quantities  $\alpha$ ,  $\beta$ , and  $\gamma$  are the phase, attenuation, and propagation constants, respectively. Knowing that the phase velocity  $v = j\omega \sqrt{\mu_0 \epsilon'}$  where  $\mu_0$  ( $= 4\pi \times 10^{-7} \text{ H m}^{-1}$ ) is the permeability of free space,  $\omega$  is the radian frequency, and  $\epsilon'$  is the real part of the complex permittivity  $\epsilon$  of the medium:

$$\epsilon_0 \epsilon = \epsilon_0 (\epsilon' - j\epsilon'') = \epsilon_0 \left( \epsilon' - \frac{j\sigma}{\omega\epsilon_0} \right) = \epsilon_0 \epsilon' (1 - j \tan \delta)$$

In this relation,  $\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$  and  $\sigma$  and  $\tan \delta (= \sigma/\omega\epsilon_0\epsilon'')$  are the conductivity and the loss tangent of the medium, respectively.

(a) Prove the following expressions:

$$\alpha = \frac{\omega}{c} \sqrt{\frac{\epsilon'}{2}} \sqrt{\sqrt{1 + \tan^2 \delta^2} - 1} \quad \text{Np m}^{-1}$$

$$\beta = \frac{\omega}{c} \sqrt{\frac{\epsilon'}{2}} \sqrt{\sqrt{1 + \tan^2 \delta^2} + 1} \quad \text{rad m}^{-1}$$

*Hint:* Calculate  $\gamma^2$ , which yields  $\beta^2 - \alpha^2$  and  $\alpha\beta$ .

(b) Show that the wavelength ( $= 2\pi/\beta$ ) is

$$\lambda = \lambda_0 \left( \sqrt{\frac{\epsilon'}{2}} \sqrt{\sqrt{1 + \tan^2 \delta^2} + 1} \right)^{-1} \quad \text{m}$$

**1.2.** The complex permittivity of skin, fat, and muscle at 915 MHz, 2.45 GHz, and 10 GHz, respectively, is given in the following table:

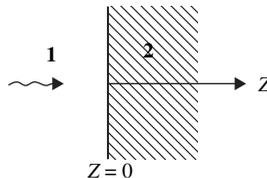
	915 MHz		2.45 GHz		10 GHz	
	$\epsilon'$	$\epsilon''$	$\epsilon'$	$\epsilon''$	$\epsilon'$	$\epsilon''$
Skin	41.5	17	38	11	31	14.5
Fat	11.3	2.2	10.9	2	8.8	3.1
Muscle	55	19	53	12.5	43	19

(a) Calculate the loss tangent and the parameters  $\alpha$ ,  $\beta$ , and  $\lambda$  for these media at the specified frequencies.

(b) The penetration depth  $\delta$  is the inverse of the attenuation constant  $\alpha$ . Find  $\delta$  for these media at the specified frequencies.

**1.3.** A plane wave is incident from semi-infinite medium 1 to semi-infinite medium 2 and propagates normal to the boundary between the two media. The reflection coefficient of a plane wave with normal incidence on a flat boundary is given as

$$\Gamma = \rho e^{j\varphi} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$



where  $\rho$  and  $\varphi$  are the magnitude and phase of the reflection coefficient, respectively. Find the reflection coefficients of the air–skin, skin–fat, and fat–muscle interfaces at the specified frequencies of problem 1.2.

- 1.4.** The electric field intensity of the plane wave inside medium 1 of problem 1.3 is given by  $E = E_{01}^+ e^{-\gamma z} + \Gamma E_{01}^+ e^{\gamma z}$ . The first term characterizes the wave propagating in the  $+z$  direction, and the second term characterizes the wave propagating in the  $-z$  direction. The interface determining the reflection coefficient  $\Gamma$  is located at  $z = 0$ . Show that the absorbed power in the unit volume is

$$P = \frac{1}{2}(\sigma |E_{01}^+|)[e^{-2\alpha z} + \rho^2 e^{2\alpha z} + 2\rho \cos(2\beta z + \phi)]$$

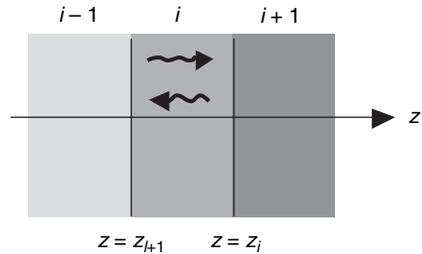
- 1.5.** Assume that a wave with  $E_0(z = 0^-) = 200 \text{ V m}^{-1}$  is propagating in the  $z$  direction inside a fat medium and is incident upon a semi-infinite muscle medium that fills  $z > 0$ .
- (a) Write explicit relations for the power absorbed per unit volume  $P$  in both media at 915 MHz, 2.45 GHz, and 10 GHz, respectively.
- (b) Plot the values of power absorbed as a function of  $z$  ( $-0.1 \text{ m} < z$ ).
- 1.6.** Repeat problem 1.5 for power absorbed in medium 2, assuming that medium 1 is air and medium 2 is skin. Assume that the skin is thick enough to ensure there is no reflecting wave present in it. Plot the results for  $0 < z < 0.003 \text{ m}$ .
- 1.7.** In multilayered cases, the reflection coefficient at the interface of mediums  $i$  and  $i + 1$  is

$$\Gamma_i = (Z_{\text{in}(i+1)} - Z_{0i}) / (Z_{\text{in}(i+1)} + Z_{0i})$$

where  $Z_{0i} = 120\pi / (\epsilon_0 \epsilon_i)^{1/2}$

and

$$Z_{\text{in}(i+1)} = Z_{0(i+1)} \frac{1 + \Gamma_{(i+1)} e^{-2\gamma_{i+1} d_{i+1}}}{1 - \Gamma_{(i+1)} e^{-2\gamma_{i+1} d_{i+1}}}$$

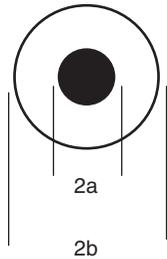


is the input impedance of medium  $i + 1$  seen in this boundary, with  $d_i$  being the thickness of medium  $i$ . Assume that a wave at 915 MHz and  $E_0^+ = 1 \text{ V/m}$  is propagating in air ( $i = 1$ ) and is incident upon three layers of skin ( $d_2 = 1 \text{ mm}$ ), fat ( $d_3 = 5 \text{ mm}$ ), and muscle ( $d_4 = \infty$ ).

- (a) Find the reflection coefficient at each interface.
- (b) Find the electric field intensity and absorbed power at the two sides of each interface and 10 mm into the muscle. Note that the electric field inside medium  $i$  can be written as  $E_i = E_{0i}^+ e^{-\gamma_i(z-z_i)} + \Gamma_i E_{0i}^+ e^{+\gamma_i(z-z_i)}$ , where  $E_{0i}^+$  is the electric field in the  $+z$  direction wave at  $z = z_i$ .
- (c) Plot the profile of the absorbed power as a function of the penetration into these media.
- 1.8.** Repeat problem 1.7 for  $f = 2.45 \text{ GHz}$  and comment on the results.
- 1.9.** Repeat problem 1.7 for  $f = 10 \text{ GHz}$  and comment on the results.

**1.10.** The dimensions of some standard 50-Ω semiflexible coaxial cables are given in the following table (dimensions in millimeters):

Outer diameter	2.20	3.58	1.19	0.58
2 <i>b</i>	1.68	2.98	0.94	0.42
2 <i>a</i>	0.51	0.92	0.29	0.13

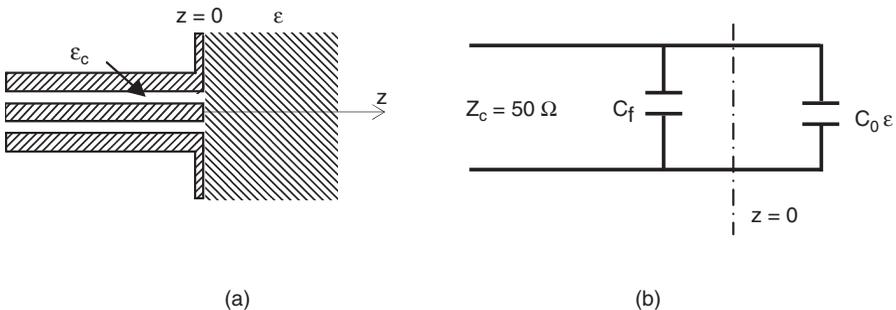


The dielectric is Teflon ( $\epsilon_c = 2.1$  and  $\tan \delta = 0.0001$ ). At 915 MHz and 2.4 GHz, find the dielectric, conductive, and total loss if the conductors are copper.

**1.11.** The 2.2-mm coaxial cable of problem 1.10 is terminated to muscle (as shown in Fig. P1.11).

An approximate model of the system is shown in Figure P1.11*b*, where  $C_f$  is the fringing capacitance on the left side of the  $z = 0$  boundary and  $C_0$  is the open-end capacitance on the right side of the  $z = 0$  boundary, where the line is air terminated. The values  $C_0 = 0.05$  pF and  $C_f = 0.01$  pF are given.

- (a) Show that when the cable is terminated to the tissue, the ratio of the real to the imaginary part of the admittance associated with the external capacitance is the loss tangent of the medium.
- (b) Calculate the input admittance and the reflection coefficient at  $z = 0$  for  $f$  values of 915 MHz and 2.45 GHz.
- (c) Calculate the input reflection coefficient seen at a source located at the input of a 100-cm coaxial line terminated at the tissue
- (d) If the source provides an available power of 10 W (50 Ω source impedance), how much is the power absorbed in the tissue?



**FIGURE P1.11** Coaxial probe terminated to lossy medium with complex permittivity  $\epsilon$ : (a) schematic of the probe; (b) equivalent circuit model.

**1.12.** Repeat problem 1.11 for fat.

**1.13.** A monopole antenna is fabricated by removing length  $l$  of the outer conductor and the dielectric of a 2.2-mm Teflon-filled coaxial cable. For a sufficiently small value of  $l$ , the impedance of the antenna in free space is approximated as

$$Z_{in}^0 = 10(kl)^2 - \frac{j60[\ln(2l/a) - 1]}{kl}$$

where  $k (= 2\pi/\lambda = \omega\sqrt{\mu_0\epsilon_0})$  is the wavenumber in free space and  $a$  is the inner conductor radius (Fig. P1.13).

(a) Use what is called Deschamps' theorem

$$Y_{in}(\omega, \epsilon\epsilon_0) = \sqrt{\epsilon}Y_{in}^0(\omega\sqrt{\epsilon, \epsilon_0})$$

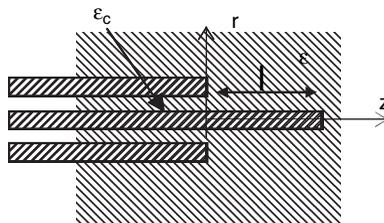
to find a general relation for  $Y_{in}$  and  $Z_{in}$ , the admittance and impedance of the monopole in the lossy medium, if the antenna is immersed in a tissue with complex permittivity  $\epsilon$ .

(b) For 915 MHz and 2.45 GHz, respectively, find the input impedance and reflection coefficient of a monopole with  $l = 10$  mm made from a 2.2 mm coaxial cable immersed in muscle.

(c) Repeat part (b) if the antenna is in a fat medium.

**1.14.** Suppose that the available power in the input to the monopole of Problem 1.13 is 2 W. Find the power absorbed in the tissue.

**1.15.** An insulated dipole with half-length  $h$  and radius  $a$  is rounded by two cylindrical concentric dielectric layers with radii  $b$  and  $c$ , and permittivity  $\epsilon_2$  and  $\epsilon_3$ , respectively. The antenna is inside a tissue medium with complex permittivity  $\epsilon_4$ . The dipole is fed by a source  $V_0^e$  placed at the middle of it (Fig. P1.15a).



**FIGURE P1.13** Coaxial probe monopole applicator immersed in lossy medium with complex permittivity  $\epsilon$ .

Given the conditions  $|k_4/k_2|^2 \gg 1$ ,  $|k_4/k_3|^2 \gg 1$ ,  $(k_2b)^2 \ll 1$ , and  $(k_3c)^2 \ll 1$  are satisfied, it is known that the current distribution on the center conductor can be written as<sup>1</sup>

$$I(z) = I(0) \frac{\sin k_L (h - |z|)}{\sin k_L h}$$

where  $I(0) = V_0^e/Z_0$  is the input current to the antenna and  $Z_0$  is the antenna input impedance with

$$Y_0 = 1/Z_0 = -(j/2Z_c) \tan k_L h$$

The parameters  $k_L$  and  $Z_c$  are complex quantities that follow the relations

$$k_L = k_{2e} [\ln(c/a) + F]^{1/2} [\ln(c/a) + n_{24}^2 F]^{-1/2}$$

and

$$Z_c = (\omega\mu_0 k_L / 2\pi k_{2e}^2) [\ln(c/a) + n_{2e4}^2 F]$$

with

$$k_{2e} = k_2 \left[ \frac{\ln(c/a)}{\ln(b/a) + n_{23}^2 \ln(c/b)} \right]^{1/2}$$

$$\epsilon_{2e} = \epsilon_2 n_{2e4}^2$$

the effective wavenumber and dielectric constant, respectively, of an equivalent dielectric for regions 2 and 3, where  $n_{2e4}^2 = k_{2e}^2/k_4^2$ ,  $n_{23}^2 = k_2^2/k_3^2$ ,  $n_{24}^2 = k_2^2/k_4^2$ , and  $F = H_0^{(1)}(k_4c)/k_4cH_1^{(1)}(k_4c)$ . Note that  $k_2$ ,  $k_3$ , and  $k_4$  are the wavenumbers in regions 2, 3, and 4 respectively, and  $H_n^{(1)}$  is the  $n$ th order Hankel function of the first kind.

A dipole is built with  $h = 20$  mm,  $a = 0.255$  mm and is covered with a dielectric layer (region 2,  $b = 0.84$  mm,  $\epsilon_2 = 2.1$ ) followed by an outer layer of plastic tube (region 3,  $c = 1.2$  mm,  $\epsilon_2 = 4$ ). The dipole is surrounded by heart tissue ( $\epsilon_4 = 60 - j24.5$  at 915 MHz).

**(a)** Find  $\epsilon_{2e}$ ,  $k_{2e}$ ,  $Z_c$ , and  $k_L$ .

**(b)** Find the input impedance of the antenna at 915 MHz.

<sup>1</sup> R. W. King, B. S. Tremblay, J. W. Strohbehn, "The electromagnetic field of an insulated antenna in a conducting or dielectric medium," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-31, No. 7, pp. 574-583, July 1983.

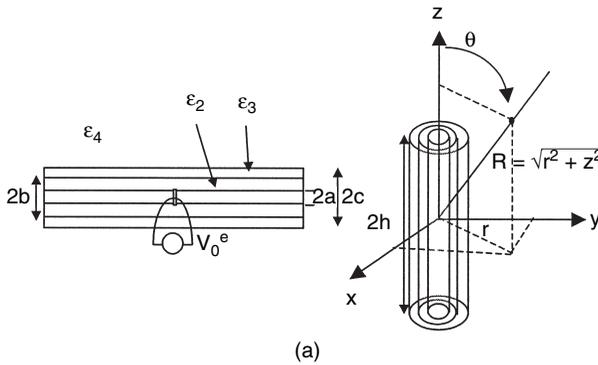


FIGURE P1.15a Insulated dipole in lossy medium.

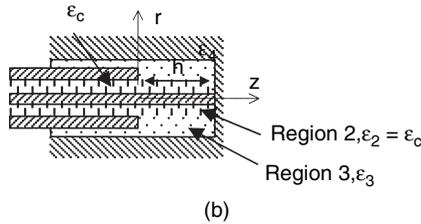


FIGURE P1.15b Catheter in lossy medium that is similar to insulated monopole corresponding to dipole of Figure 1.15a.

- (c) Plot the current distribution  $I(z)$  as a function of axial distance  $z$ .
- (d) A monopole antenna is built that corresponds to half of the above dipole (Fig. P1.15b). This monopole is fed by a 2.2-mm Teflon-filled coaxial probe ( $a = 0.255$  mm,  $b = 0.84$  mm,  $\epsilon_c = 2.1$ ). Find the input impedance of the monopole. Assume that the absence of the flange (ground plane) has a negligible effect on the monopole input admittance.

**1.16.** Repeat problem 1.15 for  $h = 31$  mm,  $a = 0.47$  mm,  $b = 0.584$  mm,  $c = 0.8$  mm,  $\epsilon_2 = 1$ ,  $\epsilon_3 = 1.78$ ,  $\epsilon_4 = 42.5 - j17.3$ , and  $f = 915$  MHz.<sup>2</sup>

**1.17.** According to the Wien displacement law, the wavelength–temperature product is a constant [Eqn. (1.44)]. In Section 1.4.3 a value of 0.0048 mK

<sup>2</sup> R. W. King, B. S. Trembly, J. W. Strohbehn, “The electromagnetic field of an insulated antenna in a conducting or dielectric medium,” *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-31, No. 7, pp. 574–583, July 1983.

is obtained. The derivation involves an approximation. It has been mentioned that a more accurate value of 0.0051 mK can be obtained without this approximation. Prove this assertion.

- 1.18.** Calculate the brightness of a blackbody radiator at a temperature of 6,000 K and a wavelength of  $0.5 \mu\text{m}$ .
- 1.19.** When the brightness is expressed in terms of unit wavelength, the wavelength for the peak brightness is not the same as when brightness is expressed in terms of unit bandwidth. The quantitative relation for the  $\lambda_p T$  product when brightness is expressed in terms of unit wavelength is obtained by maximizing (1.42) and simplifying. Prove that it yields the value 0.0029 mK [15].
- 1.20.** Show that the arc plot of a dielectric with a single relaxation time (Cole-Cole diagram, Eqn. 1.52), obtained by representing  $\epsilon''(\omega)$  as a function of  $\epsilon'(\omega)$ , is a semicircle. Verify that a given point on the semicircle corresponds to a given frequency while the summit corresponds to  $\omega\tau = 1$  and find the points for  $\omega = 0$  and  $\omega = \infty$ . Observe the disadvantage:  $\omega$  does not appear in it. Any material with a single relaxation time, characterized by  $\epsilon_s$  and  $\epsilon_\infty$  gives the same arc plot. Observe that it is therefore essential to supply the magnitude of  $\tau$  in addition to the arc plot.